

HEWLETT-PACKARD

○ **K E Y B O A R D**

VOL. 7 NO.1

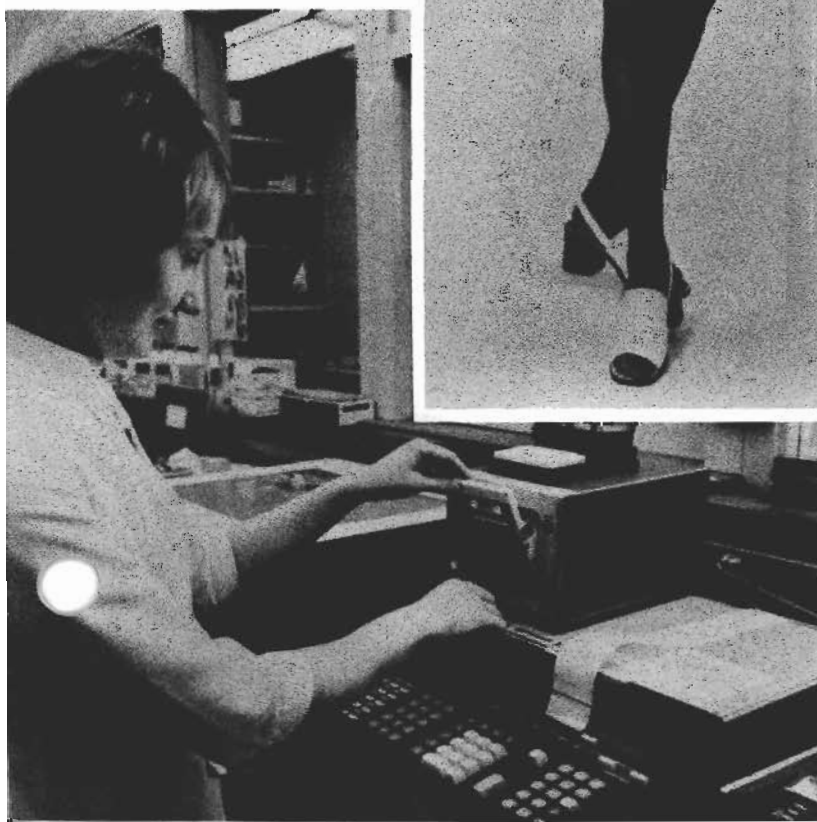
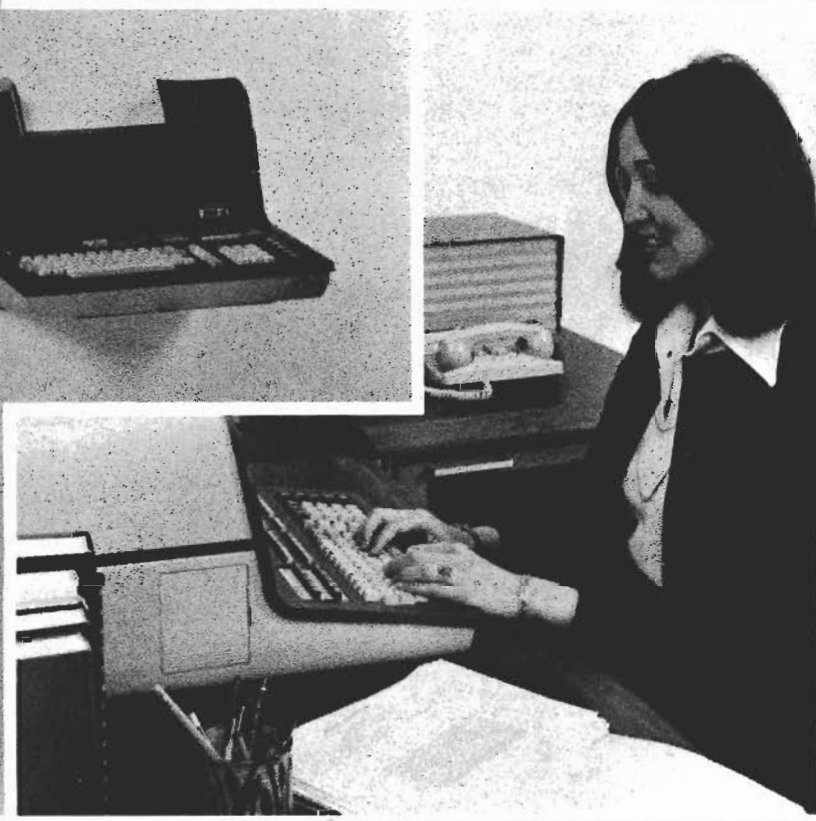
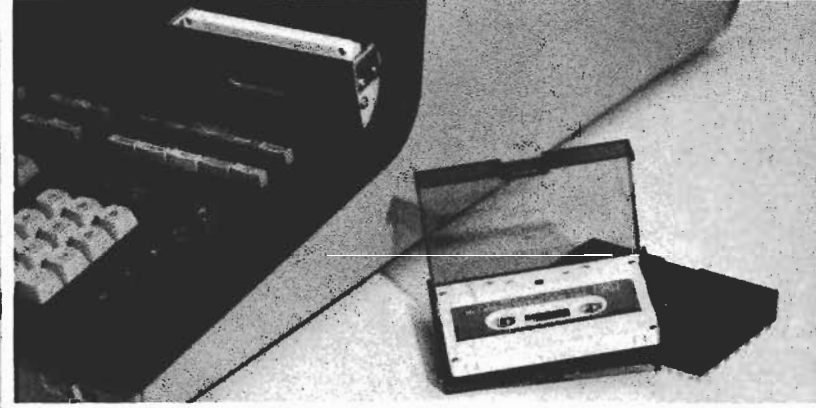
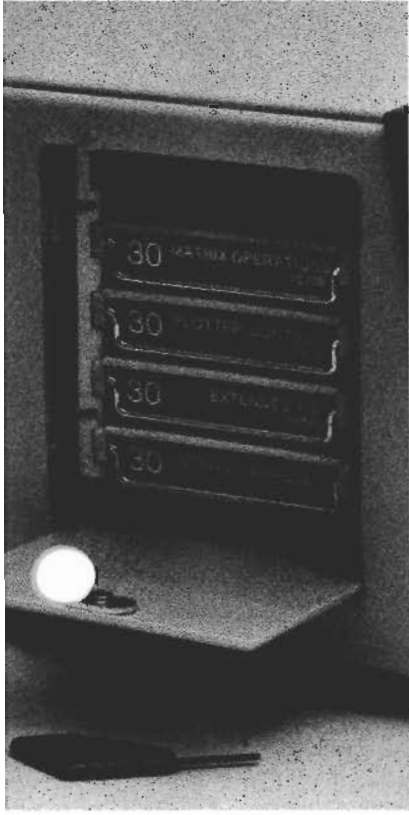


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APPLICATIONS INFORMATION FOR HEWLETT-PACKARD CALCULATORS PUBLISHED AT P.O. BOX 301, LOVELAND, COLORADO 80537

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OVERVIEW

KEYBOARD belongs to you, a user of HP desk-top programmable calculators. If you have a calculator application that would make an interesting article or a technique that would make a time-saving or accuracy-enhancing programming tip, please don't be bashful — send it in for possible publication. Contributions should be typewritten, double spaced, and accompanied by sharp black-and-white photographs 1016 mm x 1270 mm (4 x 5 inches) or larger.

The interfacing capabilities of current programmable calculators have made process control and data collection systems realizable at low cost compared to alternative solutions. Some of the possible combinations allowing a do-it-yourself approach to building systems with HP calculators are described in Cal Finn's article on page 1.

Many HP 9830 owners with project scheduling problems may find John Gassner's article on page 4 useful in automatically plotting easily updated milestone charts. This was an entry in the 1974 Calculator System Application contest.

For our readers who like to keep informed about all HP calculator products, short descriptions of the new HP-21 and HP-55 scientific pocket-size calculators are found on page 6.

A number of our readers proposed solutions to the problems of exchanging data among several HP 9820/9821 data registers without using an extra storage register. Practical approaches to solving this problem are discussed on page 8.

"The Crossroads" in this issue continues the discussion on "The Art of Science" and gives solutions to three of the problems posed in Part One (Vol. 6, No. 4).



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Calculator-Based Systems

by Cal Finn
Hewlett-Packard Calculator Products Division



Minicomputers have been used to automate repetitive tasks, such as, data collection and process control, for several years, but the complexity and cost of such systems have frightened many potential users away. There is a solution to this "all or nothing at all" dilemma.

The Hewlett-Packard 9800 Series programmable calculators solve a large number of automation problems previously reserved for computers. The ease of programming, coupled with an extensive input/output structure, allows users to assemble custom systems in their own way — without having to learn a complicated language. The portability of the calculator means that it need not be dedicated to a system but is readily available for other calculator programming.

For "do-it-yourself" type systems, Hewlett-Packard provides four interface cards that allow the majority of modern instruments to be connected to a 9800 Series calculator for control and data gathering. These cards are:

- HP 11202A — For 8-bit parallel data transfer either into or out of the calculator.
- HP 11203A — Provides for up to 9 digits of 8421-coded BCD input to the calculator. This card is designed to allow flexibility in interfacing calculators to digital voltmeters, counters, etc.
- HP 11205A — An RS-232-C compatible I/O card intended to tie teletype-like machines to the calculator.
- HP 59405A — The new Hewlett-Packard Interface Bus (HP-IB), which allows up to 14 HP-IB compatible instruments to be simultaneously connected to a 9800 Series calculator with one interface card.

The HP-IB is a unique interface system. It is bit parallel, byte serial and uses the same lines to input or output information. Hewlett-Packard has developed several instruments that are HP-IB compatible, including the HP 3490A Digital Voltmeter, HP 5345A Counter, and the HP 3495A Scanner. Also available are several accessories, such as, a 16-digit numeric display, a clock, and a digital-to-analog converter.

Using the 11202 and 11203 allows instruments that are not HP-IB compatible, but do interface to standard TTL levels (0 and +5V), to be connected to a HP 9800 Series calculator. Calculator-based systems have found acceptance in the fields of instrumentation, medical data logging, quality assurance, incoming inspection, and many others.

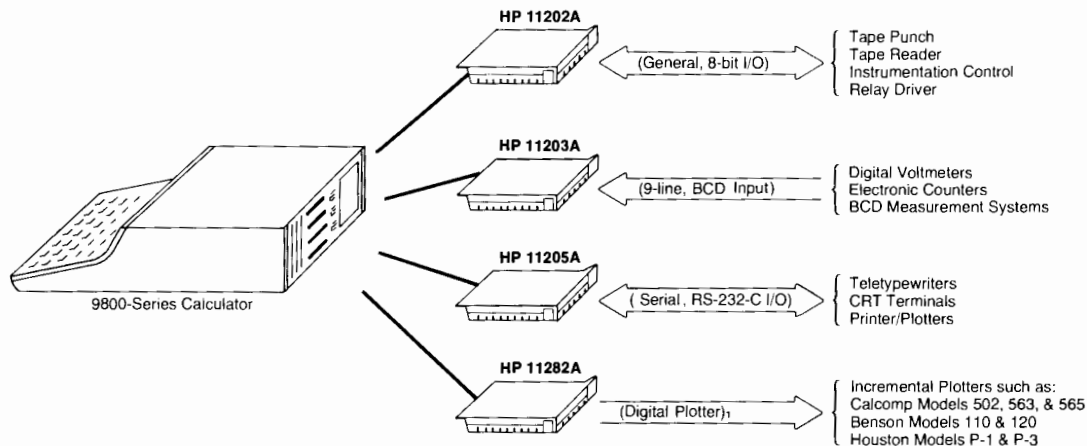
The advantages of using a calculator-based system are many. Programs to operate the system can be readily stored on magnetic cards (HP 9810A and HP 9820A) or on tape cassettes (internal on the HP 9821A and HP 9830A). Using the user-definable capabilities of the calculators (9820, 9821, and 9830), manual operation of the system can be accomplished through single keystrokes. The calculator can process data and store it on tape, plot results on a plotter, or issue reports on a printer.

The friendliness of the keyboard, the versatility of the I/O structure, the power of the built-in language, and the low cost make calculator-based systems attractive for custom systems. To aid potential system builders, HP offers "Selecting an HP Calculator Interface." Please use the reply card in this issue of *KEYBOARD* to order this publication.

COMPARING 9800 SERIES CALCULATORS

	HP 9810A	HP 9820A	HP 9821A	HP 9830A
Language	RPN ¹	Algebraic	Algebraic	BASIC
Keyboard	Key per Function	Key per Function	Key per Function	Alphanumeric
ROM Size (bytes)	5K to 11K	8K to 14K	8K to 14K	15K to 31K
RWM Size Available to User	500 to 2036 Keystrokes 51 or 111 Data Registers	1384 to 11624 Bytes	1336 to 11624 Bytes	3520 to 15808 Bytes
User Definable Keys or Functions	Optional — Single-key Subroutine	Optional — Single-key Subroutine or Function with Parameters	Optional — Single-key Subroutine or Function with Parameters	Standard — Subroutine or Function with One Parameter
Recording Device	Magnetic Card (cassette optional)	Magnetic Card (cassette optional)	Cassette Standard	Cassette Standard (disc memory optional)
Display	3-Register Numeric LED	16-Character Alphanumeric LED	16-Character Alphanumeric LED	32-Character Alphanumeric LED
Primary Printer	Optional 16-Column Alphanumeric	Standard 16-Column Alphanumeric	Standard 16-Column Alphanumeric	Optional 80-Column Alphanumeric

¹ Reverse Polish (Lukasiewicz) notation and an operational register stack provide the most efficient way known to evaluate mathematic expressions.



¹The Digital Plotter Interface can be used with the HP 9830 only.

Contest Winners Receive Awards

The two U.S. winners of the 1974 Calculator System Application Contest chose HP-45 pocket-size calculators for their prizes. Again, we extend congratulations to Dr. Alan Wray and Mr. Keith Mitchell for their winning entries to the contest and our sincere thanks to the many people who submitted entries. We appreciate the response of our readers and the high quality of the entries.



Mr. Keith Mitchell (left) is shown receiving congratulations from Rodger Settergren of the Bellevue, Washington, sales office for his article, "Airplane Stability Augmentation System Design" for the 9830.



Dr. Alan Wray (right) receives congratulations from John Regan, Calculator Regional Sales Manager in Atlanta, Georgia, for his entry, "Chess-Playing Program for the 9810A."

HP 9830 as an Aid in Tracking Project Milestones

by John J. Gassner



INTRODUCTION

A major concern of any engineering enterprise or other project-oriented organization lies in tracking the progress of on-going business efforts. Such an effort may include the construction of a large dam for hydroelectric power generation, or it may deal only with a plan for rearranging office space, including provisions for movement and placement of desks, telephones, and associated facilities.

Regardless of the situation, it is important that a plan be formulated for carrying out the effort, and it is equally important that a means be devised for tracking its execution. The Hewlett-Packard 9830A Calculator provides a time-saving tool for preparation of milestone charts useful in detailing information on project execution. The Project Milestone Program provides the user with the capability to:

- Automatically prepare schedules for planning of project milestones,
- Automatically modify and update these schedules to reflect milestone completion and/or slippages as often as required.

EQUIPMENT REQUIRED

The basic equipment used for tracking project milestones includes the HP 9830A Calculator with extended memory (3,808 16-bit words), the HP 9866A Printer, the HP 9862A Plotter, and the String Variables ROM. As currently available, the total program requires approximately 3,000 words for storage and is retained on the magnetic tape cassette used with the 9830. The size of the program can, of course, be modified in accordance with the needs and desires of the individual user. The calculator is used entirely in a conversational mode and pertinent data on milestones are entered from the

keyboard, although simple modifications can be made to allow milestone data entry via magnetic tape (along with the basic elements of the program itself) or through use of the HP 9869A Card Reader.

USE OF THE PROGRAM

The Project Milestone Program is constructed so as to provide information not only to the project manager or engineer responsible for the effort, but also to his superiors and associates who must track execution of the effort. It often seems that vital information on a project is most inaccessible when it is most urgently needed. A chart must contain information on the state of schedule completion and tell the inquirer whom to see for additional information. For this reason, the name, mail stop, and telephone number of the person responsible for compilation of the chart is plotted on the chart itself. Other vital information, such as, date and number of the chart, is also included.

The program requires little familiarization with the 9830 and, therefore, is ideally suited for use in most offices wherein many people have little time for learning the more detailed aspects of programming. The program is initiated with the RUN-EXECUTE commands and immediately enters into dialogue with the user by requesting information on:

- The project title,
- The assigned project number, if any,
- The division or company responsible for the effort,
- The responsible individual, his mail stop or office symbol, and telephone number,
- The changes made to the milestones themselves since the last report.

Each time an item of information is required, the request for that item appears on the thermal printer. For those items that may be long and cumbersome to key in and thus represent possible sources of error, the program outputs the entry on the printer, simultaneously asking the user whether or not it was correctly entered. If not, the user types NO and is given an opportunity to correct his mistake. After all preliminary data are entered, the information is plotted using the 9862 Plotter; a legend explaining symbols used on the chart is automatically prepared in the upper left-hand portion of the plotting area.

MILESTONES AND ASSOCIATED DATES

The Milestone Chart Program provides for entry of up to 20 milestones per page ranging from 0 to 48 months in duration (these limits have been established to assure that milestones are not unduly crowded together on the page). If more than 20 milestones are required, an additional page may be used. After entering preliminary information as noted above, the user is requested to

specify both the beginning and concluding month and year for the overall project, and the time frame is then plotted in monthly increments. Each year of the project is separated from the next by a vertical bar. The user is then ready to enter the first milestone to be depicted, along with the planned initiation and completion dates for that milestone. Again, he is given a chance to review his entry to be certain it is correct before plotting commences. This information is then plotted on the chart and an unshaded horizontal bar is used to depict the planned duration of the milestone.

The user is then asked if any effort has been conducted on that milestone. If so, the beginning date and the most current date on which associated effort was performed are entered. In addition, the user is given an opportunity to state whether or not the effort on the activity represented by that milestone has been completed. Actual work is then depicted on the chart as a shaded horizontal bar. If the milestone is completed, an inverted isosceles triangle is used to denote this fact. Each milestone is constructed similarly until all have been entered and the chart is completed. A sample of the completed milestone chart is presented in Figure 1. Actual questions as they appear on the 9866 Printer are shown in Figure 2.

SUMMARY OF BENEFITS

The Milestone Chart Program, when used in conjunction with specified Hewlett-Packard equipment, provides the user with a method for making real-time changes to schedules on the status of an on-going project. The program eliminates preparation of a preliminary chart or schedule draft; submission to a typing pool or secretary for typing (with attendant problems of turnaround time); correction of a typed draft and resubmission for final typing; and the need for drafting tape or stenciled symbols to complete the chart. Instead, an individual unskilled in the use of even the BASIC language may sit down at the 9830 and in a short time prepare a project milestone schedule truly professional in appearance.

CURRICULUM VITAE

Mr. Gassner is employed in the Composites Division of the Organic Materials Laboratory, U.S. Army Materials and Mechanics Research Center, Watertown, Massachusetts. Formerly he was a Senior Manufacturing Technology Engineer with the U.S. Army Aviation Systems Command, St. Louis, Missouri. He was previously affiliated with the U.S. Army Logistics Management Center (ALMC) Training Program at Texarkana, Texas.

Mr. Gassner, a native of Denver, Colorado, holds a Bachelor of Science in Engineering (BSE) from The Catholic University of America, Washington, D.C., and a Master's degree in Industrial Engineering from Texas A&M University, College Station, Texas.

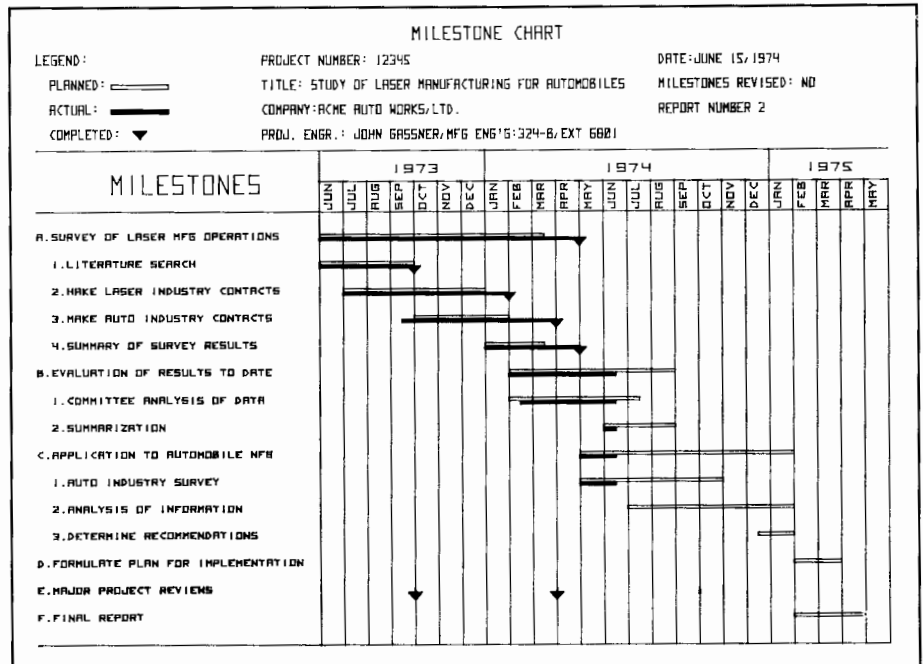


Figure 1. A milestone chart. The inverted isosceles triangles indicate completed milestones.

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ENTER PROJECT NUMBER OR ALPHANUMERIC DESIGNATION.
ENTER PROJECT TITLE; FIFTY CHARACTER LIMIT.
HERE'S WHAT YOU GAVE ME! IF OKAY, TYPE 'YES'. IF NOT, TYPE 'NO'.
STUDY OF LASER MANUFACTURING FOR AUTOMOBILES
FINE! ENTER DATE OF THIS CHART.
ARE ACTUAL MILESTONES REVISED? YES OR NO?
WHAT IS REVISION NUMBER OF THIS CHART (I.E., 2,3,ETC.??)
ENTER NAME, MAIL STOP, PHONE # OF PERSON RESPONSIBLE FOR CHART.
(E.G., JOHN GASSNER, MFG ENG - 42012-A, EXT 6601)
WHAT IS NAME OF YOUR COMPANY OR DIVISION?
HOW MANY TOTAL MILESTONES DO YOU HAVE?
STATE THE MONTH AND CALENDAR YEAR BEGINNING THE CHART (E.G. 7,73)
NOW DO THE SAME FOR THE MONTH AND YEAR ENDING THE CHART.
NOW YOU'RE READY TO LABEL MILESTONES.
TYPE IN MILESTONE 1 TITLE; 35 CHARACTERS ONLY; INCL. BLANKS.
HERE IS WHAT YOU GAVE ME. IF CORRECT-TYPE 'YES' IF NOT, TYPE 'NO'.
A. SURVEY OF LASER MFG OPERATIONS
IS THIS MILESTONE PERIODIC?
TYPE THE PLANNED MO & YR FOR INITIATION AND FOR COMPLETION
OF THIS MILESTONE (E.G. 4,73 3,74)
HERE IS WHAT YOU GAVE ME:
  PLANNED INITIATION DATE: 6 - 73
  PLANNED COMPLETION DATE: 3,5 - 74
IF CORRECT, TYPE 'YES'; IF NOT, TYPE 'NO'.
HAS ANY ACTUAL WORK BEEN DONE ON THIS MILESTONE?
ENTER THE ACTUAL MONTH AND YEAR FOR THE BEGINNING DATE OF WORK AND
THE ENDING DATE OF WORK (E.G. 4,73 - 6,73) -- NOTE: THE ENDING
DATE OF WORK SHOULD CORRESPOND TO THE DATE OF THIS CHART
IF THE MILESTONE IS NOT YET COMPLETE.
HERE IS WHAT YOU GAVE ME:
  BEGINNING WORK DATE: 6 - 73
  ENDING WORK DATE: 5 - 74
IF CORRECT, TYPE 'YES'. IF NOT, TYPE 'NO'.
IS THE MILESTONE COMPLETED?
TYPE IN MILESTONE 2 TITLE; 35 CHARACTERS ONLY; INCL. BLANKS.
  
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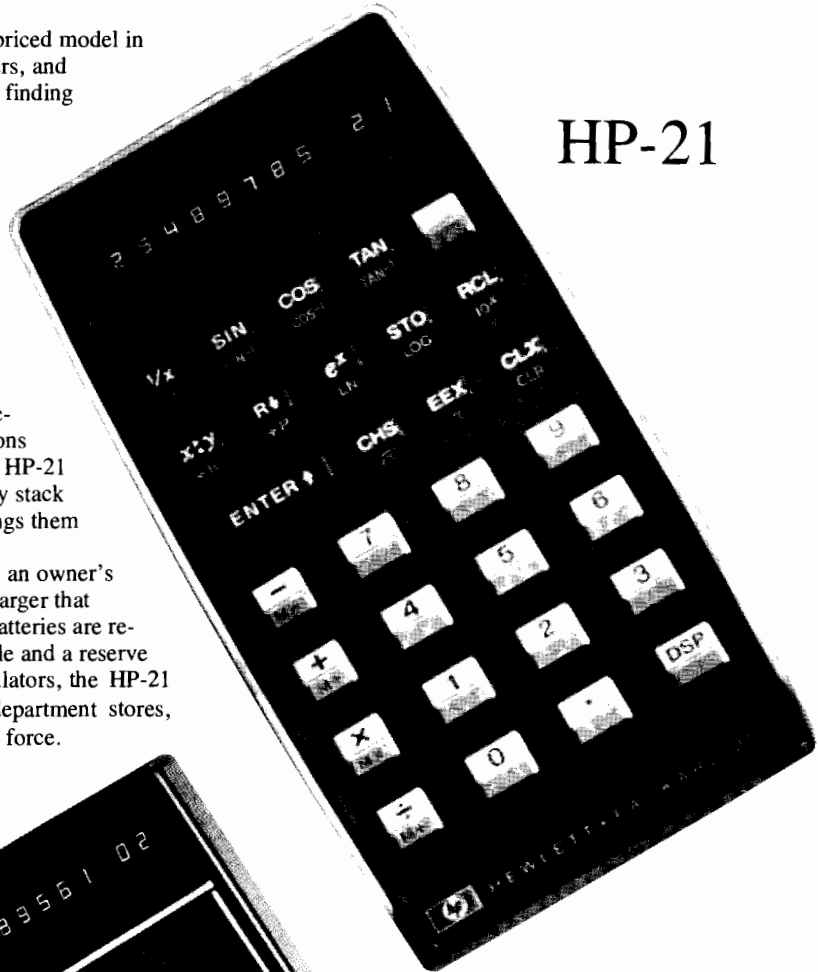
Figure 2. A 9866 printout of questions asked in the Milestone Chart Program.

Two New Pocket Calculators

The new 6-ounce HP-21, the smallest and lowest priced model in HP's line, is designed primarily for scientists, engineers, and students. Scientific calculators like the HP-21 are also finding increasing use in marine and aircraft navigation, surveying, medicine, and education. The HP-21 has all of the trigonometric and logarithmic functions of the HP-35, and the user can calculate in either degrees or radians. It also converts from polar to rectangular coordinates and vice versa, formats and rounds the display in either fixed or scientific notation, and performs register arithmetic (+, -, x, ÷) using the contents of its single addressable memory.

The HP-21 has 5 fewer keys (30) than other HP pocket models, but, since several keys serve dual functions, it is able to perform more functions and operations than the HP-35. Like other HP pocket calculators, the HP-21 has the company's RPN logic system with a 4-memory stack that holds intermediate answers and automatically brings them back when needed in a calculation.

The price is \$125 (domestic U.S.) and comes with an owner's handbook, soft carrying case, and an AC adapter/recharger that allows the calculator to be operated on AC while its batteries are recharging. Optional accessories include a security cradle and a reserve power pack with batteries. Like other HP pocket calculators, the HP-21 will be sold through leading college bookstores and department stores, by direct mail, and through the Hewlett-Packard sales force.

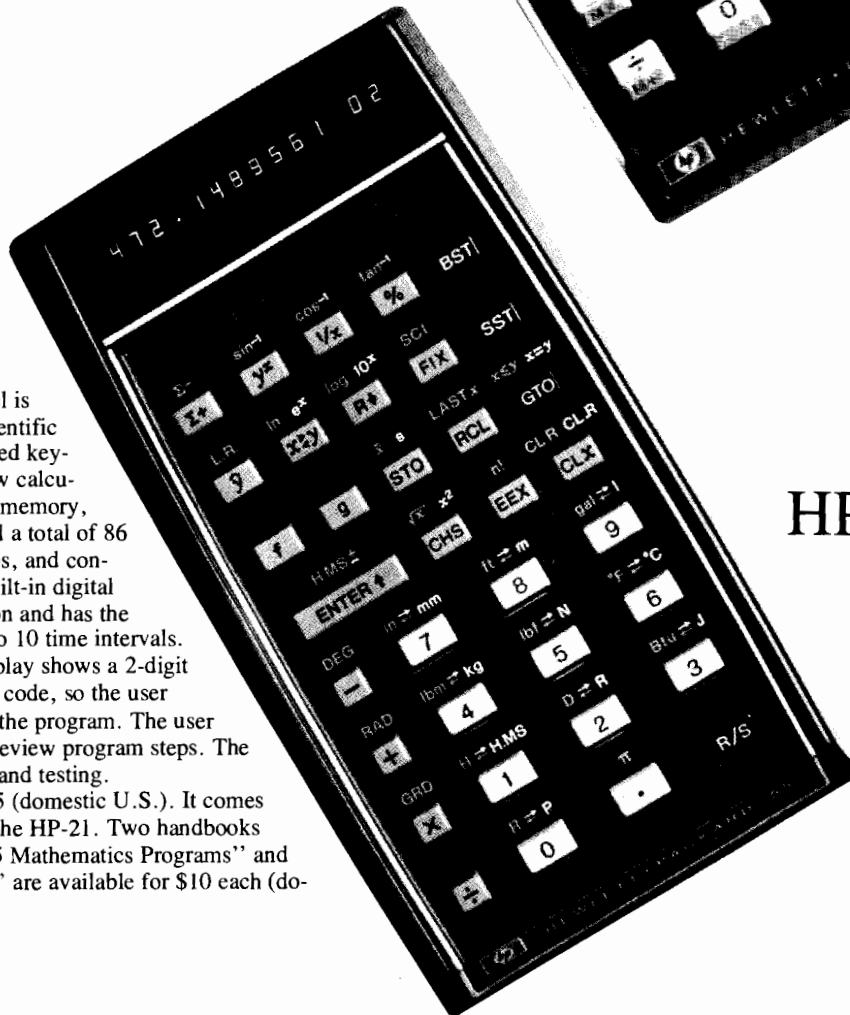


HP-21

Another new pocket model is the HP-55 programmable, scientific calculator. It features simplified key-stroke programming. This new calculator has 49 steps of program memory, 20 addressable memories, and a total of 86 keyboard functions, operations, and conversions. It also contains a built-in digital counter with 100-hour duration and has the ability to store and recall up to 10 time intervals.

In program mode, the display shows a 2-digit line number and a 2-digit key code, so the user always knows the position in the program. The user can easily correct errors and review program steps. The HP-55 also allows branching and testing.

Price of the HP-55 is \$395 (domestic U.S.). It comes with the same accessories as the HP-21. Two handbooks containing programs, "HP-55 Mathematics Programs" and "HP-55 Statistics Programs," are available for \$10 each (domestic U.S.).



HP-55

THE Crossroads

THE ART OF SCIENCE — Part 2

by John Nairn, PhD
Hewlett-Packard, Calculator Products Division

"Mighty are numbers, joined with art resistless."

EURIPIDES

To those who do not understand modern electronic calculators and computers, they may inspire awe, fear, respect, apathy, or a host of other human emotions. Purely and simply, however, the calculator is a tool. A skilled craftsman uses many tools, and must be familiar with all of the tools at his disposal and know the correct tool to use for the job at hand.

For the professional mathematician, the traditional tools have been the methods of mathematical analysis. Many varieties of these tools have been developed over the centuries, and much work has gone into honing these tools to a razor-sharp edge. The mathematician spends many years learning the existence and use of these tools, so that he may use them to advance the frontiers of mathematics and thereby develop new and even more powerful tools of analysis.

With the advent of electronic calculators and computers, mathematicians quickly recognized these machines as a new and potentially powerful addition to the mathematical tool kit. Recent decades have seen computers and traditional analysis used together to tackle problems that had resisted analysis alone.

Also, these calculating machines have advantages to offer other branches of the sciences. Mathematics has been called the queen of the sciences because other disciplines express their concepts in her language and use her methods to arrive at solutions to the problems that arise in these branches.

Physicists, chemists, engineers, etc., need to know a great deal of background for their own discipline and may not have the time to become acquainted with many of the tools of mathematical analysis. As a result, when they require a solution for a particular problem, they must either be familiar with the mathematical technique needed to solve it, take time out to learn the technique, find a consultant who knows the technique, or solve the problem by some means other than that normally used by someone familiar with the tools of mathematics. It is in this last category that electronic calculators can be of great value.

In Part One of "The Art of Science" presented in *KEYBOARD* (Vol. 6, No. 4), I gave seven problems designed to exemplify the use of calculators in problem solving. The first problem illustrates just the point I am making here. The problem of Harold's army can be expressed as follows: Since he marched his men in 13 square arrays, we have $N = 13y^2$. In battle formation, he had to join his men to form one large square array; thus, $x^2 = N + 1$. This gives

$$x^2 - 13y^2 = 1. \quad (1)$$

Since x and y represent numbers of men, we are interested only in solutions of equation (1) for which x and y are integers. A mathematician would recognize equation (1) as a form of Pell's equation and would reach into his tool box for a technique called Diophantine analysis to solve this equation for its integral solutions. Someone unfamiliar with this technique could try other methods to reach a solution, with more or less success depending on the method tried, his own mathematical skill, and some degree of luck. If this particular problem were of immediate importance and we were not interested in learning the general technique just now, a solution could easily be obtained by using the calculator as a tool.

By rearranging equation (1) to the form $x = \sqrt{13y^2 + 1}$, we see that we are looking for an integer y such that $\sqrt{13y^2 + 1}$ is itself an integer. We could then program the calculator to try all values of y ($y = 1, 2, 3, \dots$) until we found one for which x is an integer. Doing this we find that when $y = 180$, $x = 649$, and thus $x^2 - 13y^2 = 421201 - 13 \cdot 32400 = 1$. Therefore, the number of men in Harold's army was $N = 13y^2 = 421,200$ men.

Such an attack on a problem is usually referred to as a "brute-force" method. We looked for no property of equation (1) that might lead to a simpler problem or reduce the number of cases we need to try. Fortunately, in this problem the brute-force method worked and we found a solution.

Even the brute-force method can require some analysis. We also could have solved the Harold's army problem by finding a solution such that $y = \sqrt{(x^2 - 1) / 13}$ is an integer, tested for $x = 1, 2, 3, \dots$. In this case we would have had to try 649 cases before finding a solution, rather than 180 cases using the other method.

The problem of the sailors and coconuts is an example of a problem where caution must be employed in using brute-force techniques. The problem may be expressed mathematically using eight equations to represent the eight divisions of coconuts.

$$\begin{aligned} N &= 7A + 1 \\ 6A &= 7B + 1 \\ 6B &= 7C + 1 \\ 6C &= 7D + 1 \\ 6D &= 7E + 1 \\ 6E &= 7F + 1 \\ 6F &= 7G + 1 \\ 6G &= 7M + 1 \end{aligned}$$

Eliminating A through G in the above equations, we are left with

$$279936N - 5764801M = 4085185. \quad (2)$$

Again, a mathematician would recognize this as a linear Diophantine equation whose solutions are found using a mathematical tool related to continued fraction expansions. If we choose to solve this problem using a calculator and brute-force methods, we would search for an M such that $N = (4085185 + 5764801M) / 279936$ is an integer. To show why brute-force must be used with caution, I will have to cheat and give you the answer now. It is $M = 279935$, which gives a value for N of 5764795 (very industrious sailors, indeed!). Over a quarter of a million values of M would have to be tested to get the result. What makes matters worse, before this value is reached, the term $5764801 \cdot M$ will generate a 13-digit number. Unless the calculator or computer used is accurate to more than 12 decimal places, as soon as M exceeds 173466 the last decimal place is lost and the program will either accidentally yield an erroneous answer or never find a solution.

Since the brute-force method failed, we need to back off and try a little more analysis. Consider equation (2) in the form

$$AN - BM = C. \quad (3)$$

Since $A(N + tB) - B(M + tA) = AN + ABt - BM - ABt = C$, we see that, if (M, N) is a solution of equation (3), any multiple of A added to M also gives a solution. Indeed, for every value of t ($0, \pm 1, \pm 2, \dots$) a solution is obtained. The problem has an infinite number of solutions in integers. Any solution for which $M + tA > 0$ would solve our original problem (some value of t giving the smallest positive solution). But what of the cases where $M + tA < 0$? (Remember, t can be negative.) Physically, this makes no sense, for we can hardly allow negative coconuts! However, since equation (3) had no solutions in small positive integers, maybe it has a solution in small negative integers, from which the positive solutions can be obtained. Remember, we only need one solution (even a negative one) to get all of them. We set our program to work again, this time removing the bias we gave it concerning the absurdity of negative coconuts; and, sure enough, almost before our finger comes off the RUN key, we find $N = -6$, $M = -1$ as a solution. Using this as our base solution, we find a value of $t = 1$ gives the first positive solution of $M = 279935$ and $N = 5764795$, the original number of coconuts.

This problem shows how the interplay between analysis and computation can be used to tackle a problem that stands more rigidly before either method alone.

Does this mean that the calculator is the ultimate weapon in problem solving? At this time a serious drawback to computer solutions to problems is the necessity of formulating the problem in the language of the machine. In the previous examples, it was not difficult to reduce the problems to equations for which a properly programmed calculator could find solutions. This is not, however, always an easy task.

The problem of the truck crossing the desert may be solved as follows: Let 500 miles be one unit of distance (i.e., the distance the truck can travel on one tank of gas). With two tanks of gas, the truck can cross 1 and 1/3 units in the following way. The truck goes out 1/3 unit, leaves 1/3 tank of gas, and returns. After refilling, he drives to the first cache, picks up the 1/3 tank (his tank is now full), and drives one more unit, for a distance of 500 (1 + 1/3) = 667 miles. With three tanks of gas, he can drive out 1/5 unit, leave 3/5 tank, and return. Repeating this once more leaves 6/5 tanks at the cache. After a third fill-up, he drives to the cache. The 6/5 tanks there, plus the 4/5 tank in the truck, leaves him at the cache with two tanks of gas. But we have seen that two tanks will allow him to go 1 + 1/3 units. Therefore, three tanks will take him 1 + 1/3 + 1/5 units. In a similar way, we can show that using N tanks of gas he can travel a distance

$$D = 1 + 1/3 + 1/5 + 1/7 + \dots + 1/(2N-1) \text{ units.} \quad (4)$$

The required distance is 800 miles, which is 1.6 units. A little arithmetic shows that four terms in equation (4) are required to exceed 1.6 units. Four tanks will take the truck $(1 + 1/3 + 1/5 + 1/7) = 1.676$ units, or slightly over 838 miles. Since he used four tanks and had about 38 miles left, he drove a total of 1,962 miles to cross the 800-mile desert. Also, since the series in equation (4) is divergent (i.e., the sum can reach any desired value if enough terms are used), a desert of any width can be crossed. However,

the amount of gasoline increases exponentially — hardly a desirable way to travel in this day of energy conservation!

Had we wanted to solve this problem on a calculator, we probably would have expended more time and effort in trying to find a symbolism in which to express the problem than the straight analysis required. Many of the unsolved problems of mathematics have not been cracked because the analytical tools to solve them have not been developed and no one has found a suitable symbolism in which to express them so they may be run on computers.

Next time, I will give the solutions for the four remaining problems posed, which deal with questions of probability, and will offer more comments on calculator approaches to problem solving and pitfalls to avoid in using these methods.

CROSSROADS REVIEW

I would like to thank the many readers who sent comments and solutions for the problems posed in the "Crossroads" article on calendar calculations. The following readers sent me programs for "Day of the Week" calculations: R. A. Hirvonen, V. I. Johannes, John W. McCaskey, Alan J. McCombe, Frederick C. Pritzlaff, Sy Ramey, Wilhelm Reuter, Chris Simpson, A. H. Van Asten, and V. Zimmerman. Jan Olsfors and A. S. Tickner sent in interesting variations on this program in which the year is reckoned from March 1. This forces any occurrence of February 29 to be the last day of the "shifted year" and eliminates the need for leap-year corrections.

Phil Penney correctly solved the century correction problem, and Frederick C. Pritzlaff solved the perpetual calendar puzzle. (Both solutions were given in *KEYBOARD*, Vol. 6, No. 5, Page 3).

Ernest M. Goldstein brought to my attention the fact that the Gregorian calendar was adopted in Europe on October 4, 1582, not October 5, as stated in the article. Otherwise, only 9 days are accounted for, instead of the 10 days that were actually dropped.

* * * * *

Solution to Data Interchange Puzzle (9820A/9821A)

The data interchange puzzle proposed in *KEYBOARD*, Vol. 6, No. 4, drew responses from the following people: D. E. Hirst, Slough, England; Richard Buerk, Buffalo, New York; Robert Morgan, Denver, Colorado; W. A. Fuhse, Buckinghamshire, England; Milton Cohen, Rancho Palos Verdes, California; Jan Joncū, Jablonec nad Nisou, Czechoslovakia; Harry Ferguson, Seattle, Washington; R. K. Littlewood, Madison, Wisconsin; S. M. Fraser, Glasgow, Scotland; K. G. Booth, Mississauga, Ontario, Canada; and Ed Hop, Hewlett-Packard GmbH, Boblingen, Germany.

Most of the solutions submitted used a pair of inverse operations (+ and -) or (* and /) to accomplish the interchange. These methods are of two fundamental types. One type uses multistatements: $AB \rightarrow A$; $A/B \rightarrow B$; $A/B \rightarrow A$. The other type uses a single statement: $A + B - (B \rightarrow A) \rightarrow B$. Both types suffer from several

possible disadvantages. One must assume that the forward function (+,*) followed by the inverse operation (-,/) restores the original number. Several (+, -) or (*, /) operations occur that are time consuming. Extension to many-variable interchange is not immediately obvious.

By far, the fastest and simplest method is given by ...

$$1A \rightarrow (B \rightarrow A).$$

Here we have exchanged A and B with no loss of accuracy for any value of the variables, using no arithmetic other than the multiplication of A by 1 to stack its value as a temporary result.

This method is easily extended to an n-register roll around exchange by ...

$$1A \rightarrow (B \rightarrow (C \rightarrow (X \rightarrow (Y \rightarrow (Z \rightarrow A))))).$$

PROGRAMMING tips

SPEEDING COUNTERS (9820A/9821A)

Howard Rathbun of Hewlett-Packard Calculator Products Division contributed this programming tip.

The following program is a straightforward method that takes 20 seconds to execute 1,000 iterations.

```
0:
0→R0←
1:
IF (R0+1→R0)≤1000
0:JMP 0←
2:
END ←
Σ11999
R1417
```

INPUTTING VARYING QUANTITIES OF NUMBERS (9830A)

Mr. Robert Hardesty, Research Chemist at DuPont in Rochester, New York, submitted this helpful tip.

Inputting of several numbers from the HP 9830A keyboard is normally controlled by the number of variables specified in the INPUT statement. Sometimes it may be desirable to allow a varying quantity of numbers to be inputted; for example, plotting routines for a variable number of curves from different data tape files.

The following schemes allow inputting any quantity (up to 20 in these examples) of numbers. When requested, the numbers are typed in, separated by commas, as usual, and are inputted as a string. The VAL statement returns the numerical equivalent of the string up to the first nonnumerical digit, a comma. This value is either used immediately or placed in an array for later use. The 9830 then searches for the first comma and resets the string equal to everything beyond the comma. The program then returns back to find the VAL of the new string. When a comma is no longer found, the loop is exited and the rest of the array can be filled with dummy zeros. N is then the number of variables inputted.

To use the numbers immediately:

```
10 DIM A#(80)
20 DISP "ENTER NUMBERS";
30 INPUT A#
40 FOR N=1 TO 20
50 X=VAL(A#)
60 REM AT THIS POINT DO THE DESIRED
   OPERATION ON THE NUMBER X
70 Z=POS(A#,",")
80 IF Z=0 THEN 110
90 A#:=A#[Z+1]
100 NEXT N
110 END
```

To accumulate the numbers in an array for later use:

```
10 DIM A#(80),B(20)
20 DISP "ENTER NUMBERS";
30 INPUT A#
40 FOR N=1 TO 20
50 B[N]=VAL(A#)
60 Z=POS(A#,",")
70 IF Z=0 THEN 110
80 A#:=A#[Z+1]
90 NEXT N
100 GOTO 140
110 FOR Q=N+1 TO 20
120 B[Q]=0
130 NEXT Q
140 END
```

The next program does the same thing in 16 seconds. The speed increases because the calculator must calculate where R0 is, whereas, it knows where A is.

```
0:
0→A←
1:
IF (A+1→A)≤1000;
JMP 0←
2:
END ←
Σ12036
R1418
```

The next program is even faster, but the reason is not so obvious. This program takes 10 seconds to do 1,000 iterations. The reason is that the calculator must use the "number-building routine" three times (for 1, 1,000, and 0) for each iteration in the preceding program, but in this program the number-building routine is used only in line 0 before entering the loop. Finding the numbers in registers is faster than creating them.

```
0:
0→A+B;1→C;1000→X
1:
IF (A+C→A)≤X;
JMP B←
2:
END ←
Σ15203
R1417
```

Finally, the last program is a bit faster — 9 seconds. This is because the two statements are replaced by one in line 1. This program also illustrates the use of a relational operator to determine whether to JMP 0 or JMP 1.

```
0:
0→A;1→C;1000→X←
1:
JMP (A+C→A)>X←
2:
END ←
Σ13856
R1417
```



**RECTANGULAR TO POLAR COORDINATES
(9820A/9821A)**

Roland Heer of Allgemeine Elektricitats-Gesellschaft in Germany references two programming tips that appeared in Vol. 6, No. 3, "Double Unary Minus" and "Rectangular to Polar Coordinates," and suggests the following technique:

By putting together several programming tips, a still simpler program line for the HP 9820A or 9821A to convert from rectangular to polar coordinates may be obtained:

```
0:
SFG 14;2ATN (Y/(
1X+(F(XX+YY)+X))
)Y;CFG 14F
```

Though it has 18 keystrokes less than the program tip given in Vol. 6, No. 3, it does exactly the same thing.

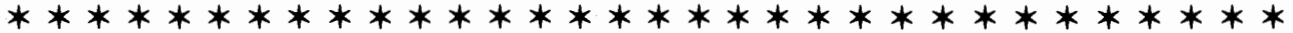
QUICK XREF (9830A)

Joe Armstrong of Hewlett-Packard offers this programming tip.

When debugging programs, it is often necessary to find the location of one or more variables (or even to see if a variable exists) within a given program. The usual procedure is to execute the XREF command found in the Advanced Programming I ROM. A printed cross reference of all the variables within a program is printed. A significant amount of time and printer paper can be used during the normal debugging of a program using this standard XREF procedure. To cross reference only the variables you are interested in, simply define these variables in the first few lines of the program. Suppose you are interested in the following variables; A, B, C, D, A(10), A\$. Simply key the following lines into your program:

```
01 A=B=C=D=0
02 A(10)=0
03 A$=" "
```

It is assumed that your mainline program starts at a line number greater than 03, and there is no common statement. Now execute the XREF command. The cross reference will print out the locations of the variables in the order shown above. After the cross reference is complete, press the STOP key to terminate the XREF command. NOTE: Be sure to delete the lines entered before saving your program.



9810A Memory Unit Salvaged

by Larry Mills, Service Manager
Ridgecrest, California Service Office

A number of months ago, there was a trailer fire at the China Lake Naval Base in California. Fully half the trailer burned, including a HP 9810A, a HP 9862A Plotter, and a HP 9865A External Tape Cassette. The Ridgecrest office, which has the service contract for China Lake, was called on to estimate the damage to the HP equipment. All three instruments were judged to be complete losses, but we recalled that the user of another 9810 at China Lake had wanted to expand the memory. Clayton Peterson, our service technician, transferred the memory unit of the burned 9810 (a fully loaded machine) to the other 9810. It worked at first turn-on and has operated without failure since.

