HEWLETT-PACKARD

KEYBOARD

VOL. 6 NO. 5

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OVERVIEW

Our panel of judges in the outside-U.S.A. branch of the 1974 Calculator System Application Contest had a difficult time selecting a winner from the 66 excellent entries. Announcement of the winner and a list of entries is found on page 8 of this *KEYBOARD*.

If you have a need for a small, light-weight card reader for either marked or keypunched cards, a solution is available. The new HP 9870A Card Reader, which acts as a remote keyboard for HP desktop programmable calculators, is described in the article on page 1 .

Multiple regression in determining real estate values is becoming increasingly important to the real estate appraiser as a versatile and practical tool. Seminars are being conducted for appraisers in the U.S.A., using Hewlett-Packard calculators. Details are given in the article on page 4 .

A hologram, or 3-dimensional picture made on film without a camera, is usually recorded using a split beam of coherent light. The article on page 6 describes the use of an HP calculator system in lieu of a laser beam to produce limited holograms which can be veiwed by illuminating a 35-mm slide of the plotted hologram with a laser beam.

Dr. James Shapiro of Texas A & M University is our guest editor for The Crossroads in this issue. We hope you will enjoy his article on Eigen-solutions of linear systems, starting on page 11.

A B Sperry

APPLICATIONS INFORMATION FOR HEWLETT-PACKARD CALCULATORS PUBLISHED AT P.O. BOX 301, LOVELAND, COLORADO 80537

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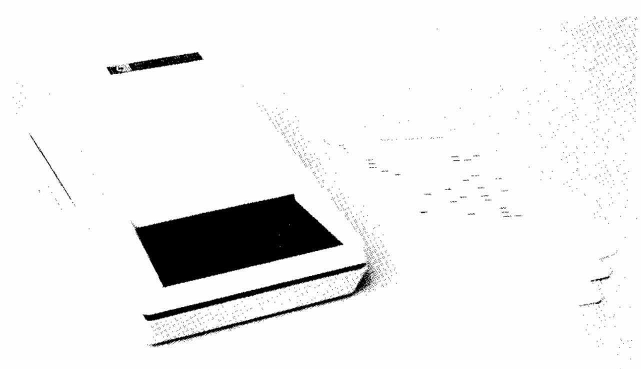
New HP 9870A Card Reader



LOW-COST, COMPACT CARD READER FOR HP PROGRAMMABLE CALCULATORS

The latest addition to the Hewlett-Packard calculator peripheral family is the low-cost, compact HP 9870A Card Reader. The new card reader is so compact and lightweight that it is virtually a hand-held unit.

The handfed card reader reads both pencil marked and punched cards. The cards are specially designed for HP 9870A and are marked in standard 96-character Hollerith code. Translation from Hollerith to various calculator keycodes is done through the interface cards. All the power necessary to operate the card reader is derived from the calculator.

The card reader is simple to operate and is essentially maintenance free. The HP 9870A operates as a remote calculator keyboard. Almost all keyboard functions can be duplicated, which makes programming and data entry less time consuming. A number of users can use the calculator without tying up the calculator keyboard. The cards used become part of the permanent records and also the source of data and program entry. The available cards are so designed that they can be punched on standard IBM keypunches.

OPERATION

Unlike most available card readers, which use reflective techniques, HP 9870A reads THROUGH the cards. Infrared light sources and photo transistors are used to sense presence of information on the cards.

Feeding a card in the reader starts the motor, which then pulls the card through. The 35-column card shown in Fig. 1 can be read in less than 2 seconds.

THE CARDS

The cards are specially designed for the HP 9870A. Figs. 1a and 1b show the front and back of the card. The same card is used for both the data and program steps. For user convenience in marking or punching these cards, Hollerith and calculator keycodes are printed on the card. The same keycodes are valid for all 9800 series programmable calculators. Fig. 2 shows a marked card illustrating Hollerith and various keycodes.

APPLICATIONS

The ability to read both marked and punched holes in Hollerith code makes the card reader an extremely useful peripheral in many applications. For example, a class of students can use the calculator more effectively by marking or punching programs and data on cards. This considerably improves the throughput of the calculator. The marked cards can be used as time cards, inventory control cards, patient history cards, job cost cards, and for many similar applications. Cards may be custom designed to suit special applications.

- In electronic design situations marked cards can be effectively used for entering various design parameters into the calculator.
- A statistician may record all his data on the marked or punched cards and perform the statistical analysis in less time and with greater reliability.
- In another situation, a market researcher may use the marked cards for entering responses to his questionnaire.
- A surveyor in the field may record all his readings on specially designed cards and on his return may use the same cards for data entry into the calculator.

Many other applications are possible for the 9870A Card Reader. Educational test scoring, billing and invoicing in warehouses, field research data, and telephone trouble reporting are a few more examples. As a remote keyboard the 9870A should prove quite useful to many more imaginative users.

The advantage of the cards is that they serve a dual purpose. They become part of the permanent records, as well as a device for data entry.

It is also possible for some large quantity users to design their own interface for the card reader to enter data into other data processing equipment.

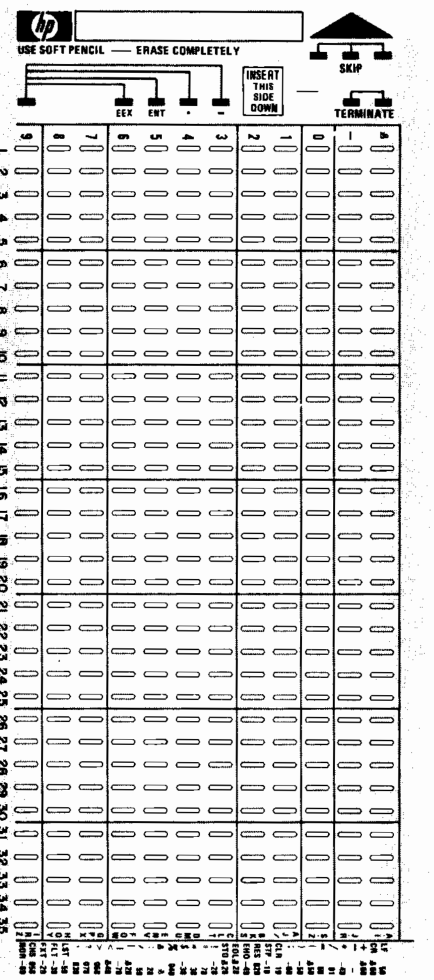


Fig 1a

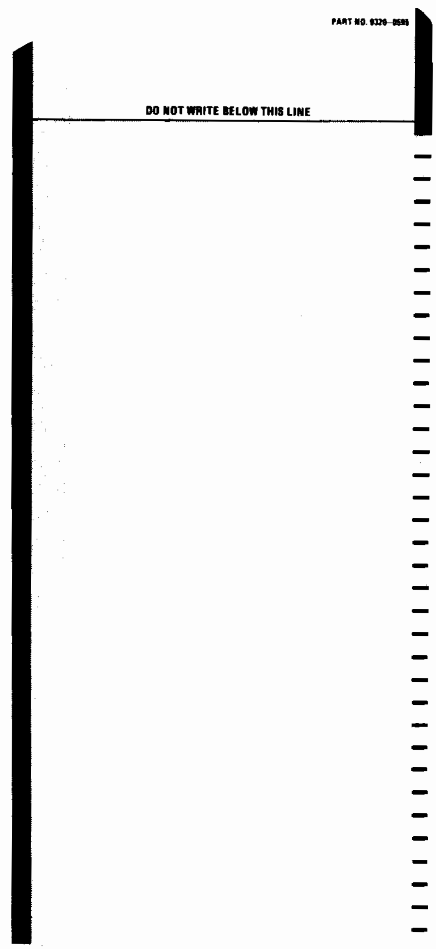


Fig. 1b

Given below is a description of codes on the left hand side of the card.

ENT: Marking or punching rows 5 and 9 in a column causes the same effect as pressing the following keys:
CONTINUE for 9810A Calculator
RUN PROGRAM for 9820A and 9821A Calculators
EXECUTE for 9830A Calculator

EEX: Marking or punching rows 6 and 9 in a column indicates ENTER EXPONENT.
NOTE: For 9810A Calculator EEX can also be obtained by marking or punching rows 0, 2, 8 in a column.

.: Marking or punching rows 4 and 9 in a column indicates a DECIMAL POINT (.) or a PERIOD(.).

—: Marking or punching rows 3 and 9 in a column indicates MINUS SIGN (-).

SKIP: Marking or punching rows 12, 11, and 0 in a column causes the Card Reader to ignore (SKIP) that column. This is a useful tool for error correction.

TERMINATE: Marking or punching in rows 12 and 11 in a column causes the further reading of that card to be terminated. Remainder of the card can then be used for comments. TERMINATE should be the last entry on the card.

ION

The card reader may be operated in either the Interrupt mode or the Data Demand mode.

Interrupt Mode

In the interrupt mode, the card reader acts like a remote keyboard. Program steps and data are coded on cards in the same sequence as if entered from the keyboard. When a card is inserted into the reader, the card reader motor starts, and the program steps or data are entered in the calculator.

Program Steps: Cards are coded with appropriate Hollerith assigned keycodes. The program instructions on the card are loaded into successive calculator memory locations.

Data: Cards are coded one digit per column. The data are entered into the calculator by using the "ENT" code.

Data Demand Mode

This mode of operation requires either a Peripheral Control or Extended I/O plug-in block installed in the calculator. The calculator starts the card reader motor when data are required. Cards coded with one digit per column may then be inserted. A delimiter (e.g., line feed, carriage return) must be coded at the end of a data entry to signal the calculator that the entry is terminated.

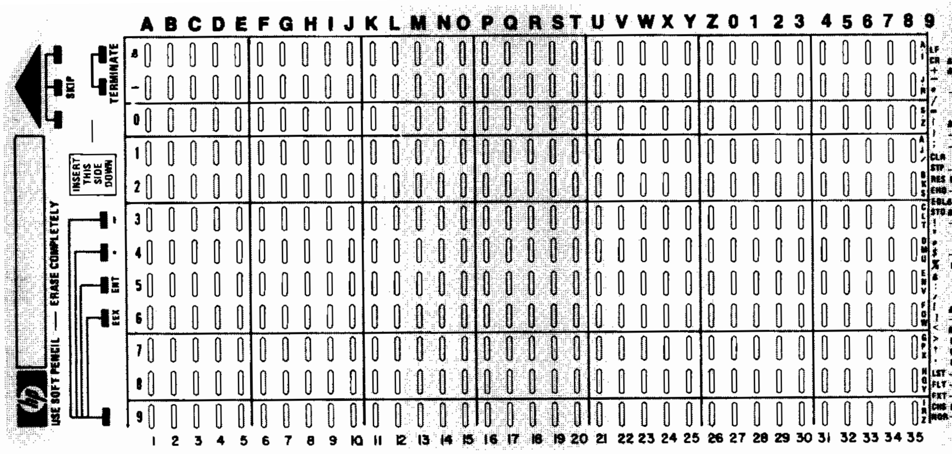


Fig. 2 Alpha and Numeric Hollerith Codes

CODI

Marking or punching a 9870A card is extremely simple. The card is marked in Hollerith code. Most of the calculator keyboard keys have a corresponding Hollerith code. The codes for A-Z and 0-9 are standard Hollerith code; some are indicated on the 9870A card, Fig. 2.

The operating manual for 9870A Card Reader shows the various other keycodes not shown on the card.

Cards can be conveniently punched on a standard IBM keypunch. A drum card may be prepared to punch the cards with greater ease. The cards are so designed that they can be punched by skipping alternate columns.

ANSWERS TO CALENDAR PROBLEMS

Following are the answers to the problems given in The Crossroads article on calculator calculations (*KEYBOARD*, Vol. 6 No. 3). Beginning with the current article, solutions to problems will be given in the second article following the one proposing them. This is to allow all readers ample time to receive their copies of *KEYBOARD* and to respond to the proposed problems.

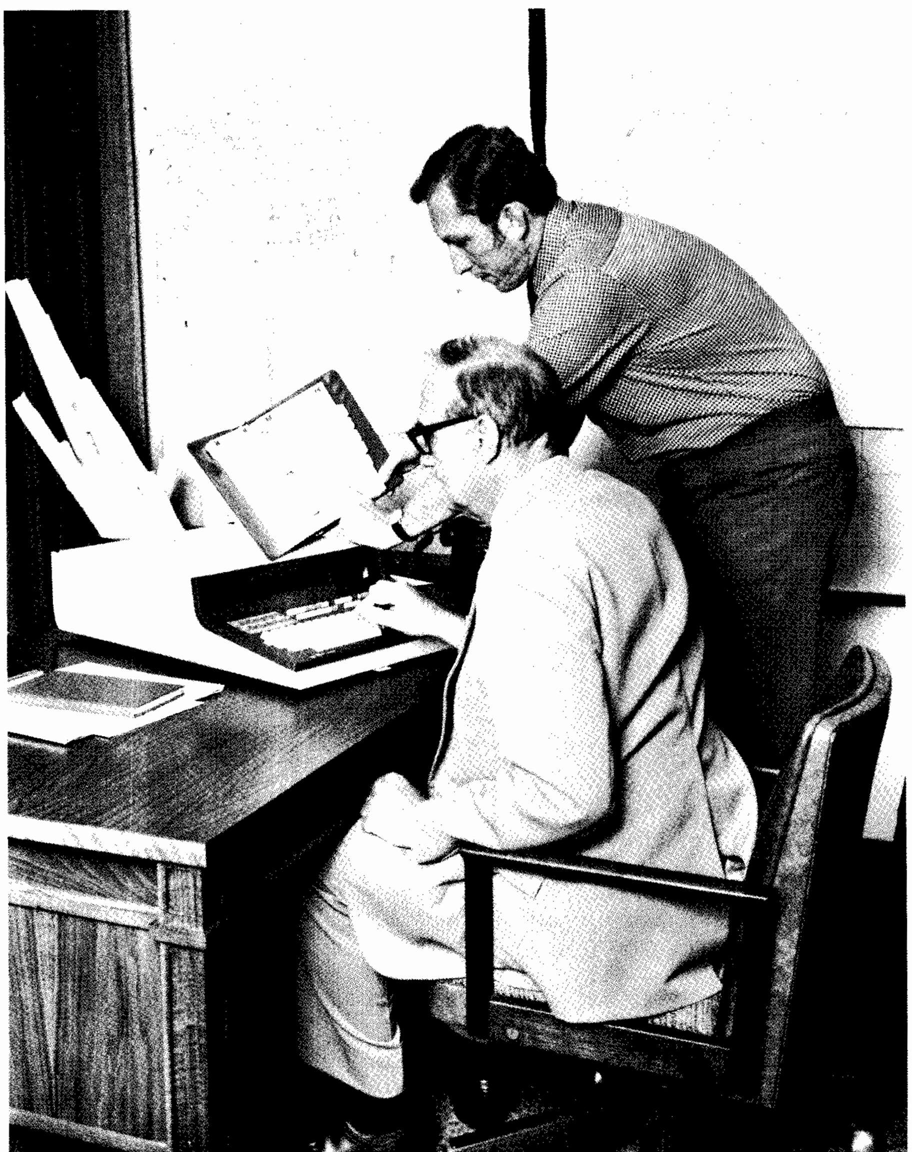
The solution to the perpetual calendar puzzle is obtained by letting one cube contain the digits 0, 1, 2, 3, 4, 5 and the other cube contain the digits 0, 1, 2, 6, 7, 8. Many readers decided that the problem had no solution, and indeed this is correct for a general number system. Our particular system of digit symbols, however, allows a nine to be represented by using the face labeled "6" and inverting it. Thus, with the configuration given above, all double digits from 01 to 31 can be represented.

The other problem was to determine the correction to be applied to the day-of-the-week formula to allow dates outside of this century. The only correct solution received so far was from Phil Penney of Utah, who observed that the correction term is a function of the century only and repeats in cycles of four centuries. His correction was in the form of a table, which can be summarized by the following algorithm.

Let the year be given by 100*C+Y, where C is the two century digits and Y is the two year digits (e.g., for 1752, C = 17; Y = 52). Let X = C (mod 4). That is, X is the remainder after any multiple of 4 is taken out of C. The correction term is then 6-2*X. For C = 17, X = 1 and the correction term is 4. Notice that for C = 19, X = 3 and the correction term is zero. This explains why the formula given in the last article works for this century without applying a century correction.

As a final epilogue to Calendar Calculations, any cynics among our readers might find it interesting to calculate the day of the week for January 2, 1900. What can you expect from a century that started on a Monday!

Valuation With MRA



MULTIPLE REGRESSION ANALYSIS: USES AND INSTRUCTION IN REAL ESTATE VALUATION

by Charles R. Blazek

Multiple regression analysis as a technique of real estate valuation has been the focus of considerable professional attention in recent years. The recognition of the power, validity, and versatility of its uses is becoming very important, not only in research and academic levels, but also in the offices of practicing professional appraisers. The method has become so important that textbooks have been written on the subject matter, articles have been written, programs for calculating equipment have been created, and professional associations have instituted instructional courses devoted to the techniques into their educational programs. In the last two areas, Hewlett-Packard has played an important role.

In 1970, the office with which I am associated gave serious consideration to devising a new concept in developing estimates of valuation for real estate. After an extensive period of time was devoted to research and study, multiple regression analysis was chosen to advance the appraisal profession, to be of assistance to the appraiser, and to justify the estimates of value in a more valid way for the client.

The several conventional analyses of market data, or approaches, use simulation behavior of market participants to estimate market value or "predict" the most probable selling price under a given market. Multiple regression analysis, on the other hand, uses statistical inference. Like the other methods of analyzing and drawing conclusions from market data, its use is at times more appropriate than at other times; and like the others, it has strong points as well as drawbacks.

In addition to the research on the subject matter, a great deal of research as to calculating equipment was undertaken. With a new concept such as multiple regression, there were areas which were considered to be of importance in the usability of the equipment in order to help encourage the members of the appraisal profession and related fields to use this concept. After a great deal of research, Hewlett-Packard was chosen because of equipment reliability and the interest shown by that company as well as the consistent research HP has made in the valuation areas.

In 1971, our office was asked by the Society of Real Estate Appraisers to present "Appraisal Implications of Multiple Regression" at the 1971 SREA Symposium in San Antonio, Texas. Hewlett-Packard was asked to support this program, and in turn would gain recognition. This was the start of a very pleasant and rewarding experience combining multiple regression and the uses of Hewlett-Packard calculators.

Since the 1971 symposium, I have been asked to develop a multiple regression seminar for the Society of Real Estate Appraisers and to present this seminar at different locations in the United States. Seminars have been presented in Salt Lake City, Utah; Troy, Michigan; Denver, Colorado; Seattle, Washington; Baltimore, Maryland; Jacksonville, Florida; Austin, Texas; and Newport Beach, California. Future seminars will be presented in Nashville, Tennessee; Columbus, Ohio; St. Louis, Missouri; Albuquerque, New Mexico; Charlottesville, North Carolina; and Miami, Florida. Many others are in the planning stages. The support and development of the material and programs are based on the uses of the Hewlett-Packard equipment. The HP-80 and 9830 calculators are used by the students in the seminars.

The Society of Real Estate Appraisers has three designations: SRA (Senior Residential Appraiser), SRPA (Senior Real Property Appraiser) and SREA (Senior Real Estate Analyst). The education program within the Society is designed to educate and inform the members and to help fulfill the requirements in order to obtain these designations. In the education program, a course designed for special applications of appraisal analysis is designed to inform the appraisers of the applications of quantitative analysis and interpretation of the results of which the multiple regression seminar is used as a prerequisite. The Society has purchased fifty HP-80's for use in the instruction of these two courses.

The Multiple Regression Seminar includes two days of instruction, in which statistical inference is used to draw conclusions about the market influencing the subject from available observations of applicable market activity.

Simple linear regression is explained to give a basic explanation as to the concept of multiple regression. The HP-80 is used to help the students in solving the equations for simple linear regression and the development of a trend line.

After solving different types of real estate problems via simple linear regression, the students are guided into the application and analysis of multiple regression. Through statistical inference we seek to draw conclusions about the market influencing the subject from available observations of applicable market activity. Further, we seek to predict market behavior as of a current date, based on noted actions in the past and the factors found to measurably influence those actions.

Multiple regression analysis is a statistical tool. In appraising, it may be used to "predict" sale price based on relationships found between market data sales prices and the several items, or variables, which influence the sales price. It is very important to keep in mind that through the use of the multiple regression, it is often possible to predict or estimate items of interest to appraisers other than price, such as multipliers, rates of return, capitalization rates, and other items that pertain to the reasons for the market reacting in a certain manner. In the course we speak of estimating or predicting the price; however, other factors may be similarly estimated. Some of the greatest potential for the use of multiple regression analysis by appraisers is in the estimation of these factors other than price.

The students are taught that when price (the dependent variable) and the amounts of various items influencing price are known for a number of comparable market data observations, the relationships of these influencing factors (independent variables) to price may be calculated. With the relationships between the dependent variable price and the independent variables or items that influence price having been calculated, it is relatively simple to find the predicted or estimated price of the subject (the unknown dependent variable), since the independent variable or factors that influence the value or price of the subject are known.

The computer is able to tell us:

1. The predicted price for the subject property.
2. How much of the variance in the price is accounted for by the various influencing factors or independent variables for the comparable observations.
3. The correlation between each independent variable or influencing factor and the dependent variable.
4. The validity of the results by means of the standard error of estimate, which is a statement of the expected accuracy of the estimate of the price.

The method of analysis by means of multiple regression has several advantages. However, the skills in the selection of variables, selection of observations or data, and quantifying influencing factors are more important and critical than in the traditional market approach. Some of the advantages are:

1. In many ways, possible judgment error by the appraiser is minimized, although his judgment is critical at key points.
2. Adjustments for differences between the subject and comparable observations are calculated from the data mathematically rather than the appraiser relying solely on judgment or experience.
3. Multiple regression can often be used as a double check on the traditional approaches.
4. It is a good test to see if an observation is truly an effective comparable.
5. Truly relevent influencing independent variables can be identified.
6. Accuracy and reliability at a desired confidence level may be calculated.

Multiple regression is a concept that is being accepted at a rapid rate by the appraisers and many other areas related to the real estate market, and has a long-lasting future. The multiple regression tool is powerful and versatile if used intelligently, but it can be abused. Multiple regression analysis undoubtedly possesses certain attributes which make it a superior value-estimating tool. It is one appraisal method which will inevitably become, if it already has not done so, an invaluable complement to the appraisal process.

Charles R. Blazek was awarded a Bachelor of Arts Degree in Business Administration with a Teaching Certificate from Colorado State University in August, 1960. He received a Master of Arts Degree in Education Administration, with an Administrator's Certificate from Colorado State University in June, 1964. Since 1969, Mr. Blazek has engaged in the valuation of residential, commercial, industrial, ranch, and special purpose properties for transfer of ownership, condemnation, mortgage loan, ad valoram tax and other purposes. Mr. Blazek is a Director and Associate Member of Colorado Chapter No. 9, Societv of Real Estate Appraisers.

Computer Holography With the 9100B

by James S. Marsh, PhD and Richard C. Smith, PhD

Computer-generated holograms have been made for almost as long a time as ordinary optically-generated holograms. Fraunhofer holograms, formed in the far field of the object, have the simplest mathematical description, since they consist of the two-dimensional Fourier transform of the holographic object, combined with a known reference wave. The numerical calculation of this transform usually requires the use of a large and speedy computer.

On the other hand, we show here that Fourier transforms of simple objects such as stick letter groups have simple analytic forms which allow computation in the limited memory of the HP 9100B, even without the extended memory. Since the Fourier transform is a linear operation, the transform of a stick letter group is just the sum of the Fourier transforms of each stick.

It is easy to show that the transform of a stick of length L, which makes an angle ϕ with respect to the X axis, and is centered at the point X_0, Y_0, is

$$F(x,y) = UL\,\mathrm{sinc}(\tfrac{1}{2}LR(x)) \qquad (1)$$

where $\mathrm{sinc}(z) = \sin(z)/z$,

$$R(x) = x\cos\phi + y\sin\phi,$$

and $U = \exp(i\,(xX_0 + yY_0))$.

The coordinates axes (X,Y) describe the object, while the parallel axes (x,y) define the Fourier transform plane.

The Fourier transform of the group of stick letters will then be simply a sum of terms similar to Eq. (1). The complexity of objects to be transformed on the calculator in this fashion is thus limited by the amount of programming space necessary to represent the various analytic transform terms. Considerable simplification of the transform sum is often possible.

Once the Fourier transform is calculated it must be plotted in a way that displays both amplitude and phase. This is accomplished by sampling the Fourier transform at the vertices of a square mesh. At the corresponding place on the plot, the pen is made to produce a vertically directed line segment whose length is proportional to the amplitude of the Fourier transform at that point. The segment is displaced horizontally from the sample point on the plot by a distance proportional to the phase of the calculated transform, with a phase of $\pm\pi$ corresponding to a displacement of $\pm$ half the mesh interval.

The holographic object is reconstructed by illuminating a 35mm slide of the plot with a parallel beam of laser light which is then allowed to pass through a long focal length lens. The reconstructed object appears in the focal plane of the lens as the first order diffraction pattern of the hologram. It is worth noting that the film exposure for the slide is quite non-critical, since linearity in processing is not a requirement for objects with high contrast, such as the hologram. Copies made with Polaroid transparency film reconstruct the original object quite nicely.

It is desirable to suppress spurious images by using a pen which makes a mark whose width is half the mesh spacing. We do this by shaving the tip of a fiber pen refill to a wedge shape and mounting it in a drilled-out HP pen.

Programming is straightforward. The Fourier transform of the object, F(x,y), is sampled at points $x_i = i\Delta$, $y_j = j\Delta$, with i and j integers and Δ the sampling interval. The sampling interval should be chosen so that most of the transform with significant amplitude appears on the plot. We have typically chosen $-20 \leq (i,j) \leq +20$ to give a plot with 40 x 40 sampled points in a square array. Choosing more sampling points gives a better reconstruction at the cost of excessive computing times. Early plots made with 120 x 120 sampling points took nearly 24 hours to complete! Most of this times goes into lifting the pen between points.

Once the Fourier transform is calculated at the sample point in the form, $F(x_i,y_j) = pe^{i\theta}$, the pen is directed to drop at the position (assuming unit scale) $x = x_i + \theta\Delta/2\pi$, $y = y_j + \rho\Delta/2$ and then to draw a thick line from this position to the new position $x = x_i + \rho\Delta/2\pi$, $y = y_j - \rho\Delta/2$. Overlap of marks (amplitude saturation) is avoided by prescaling the transform amplitude so that $\rho_{max} < 1/\Delta$. The pen is then lifted and the process repeated at the next mesh point.

Figure 1 shows a typical object which consists of ten separate line segments. The calculator-plotted hologram is shown in Fig. 2, which reconstructs to the final image of Fig. 3. Plotting time for this hologram was approximately 8 hours.

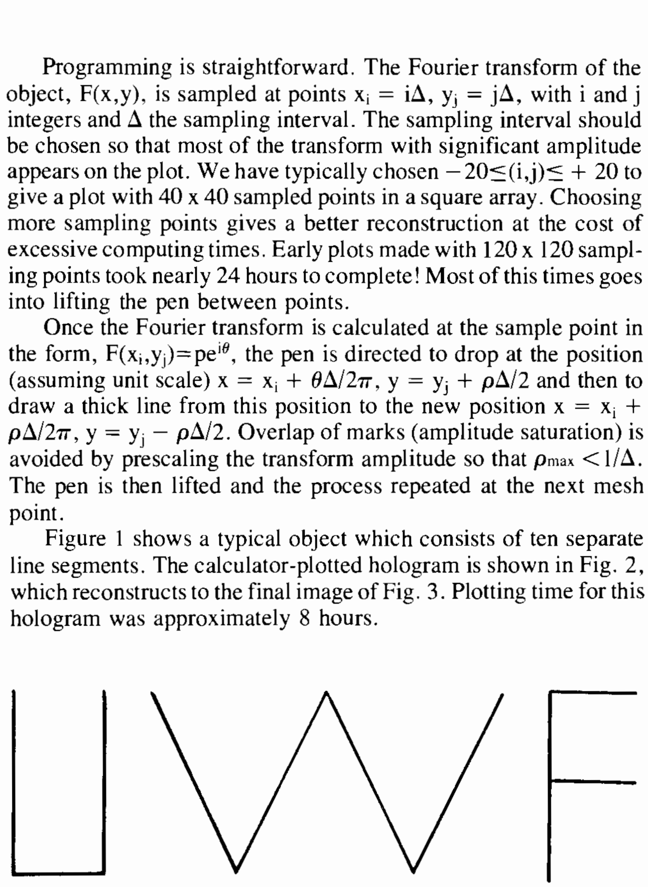


Fig. 1

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2. R.C. Smith and J.S. Marsh, "Fourier Transforms of Simple Apertures," *J. Optical Society* (to be published).
3. J.S. Marsh and R.C. Smith, "Computer Holograms with a Desk-Top Calculator," *American Journal of Physics* (to be published).

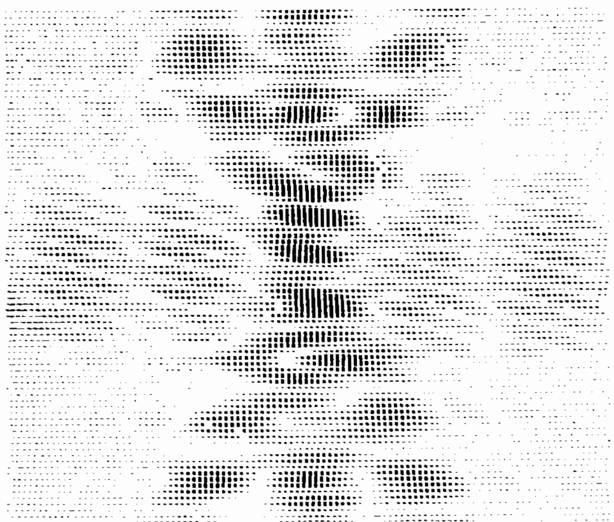


Fig. 2

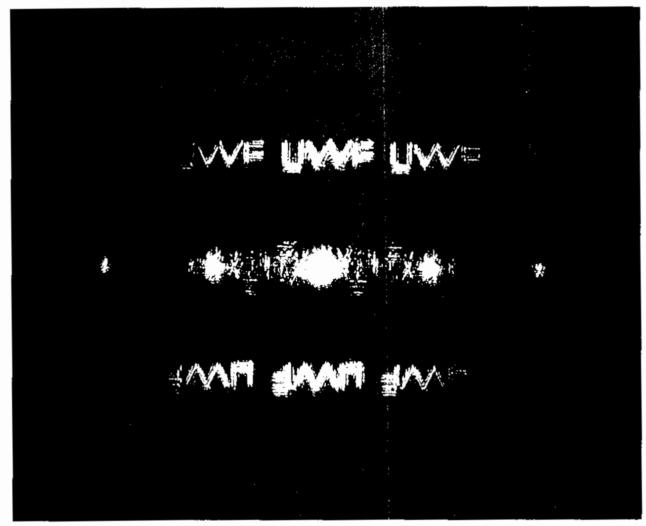
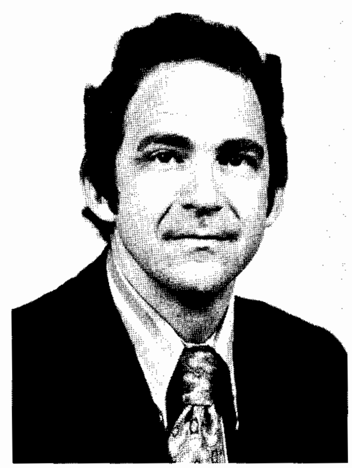


Fig. 3



James S. Marsh received his BA in Physics at Marietta College in Ohio, and a PhD in Physics at the John Hopkins University in 1966. He studied at Oxford for a year under a National Science Foundation postdoctoral fellowship. He has worked since 1969 at the University of West Florida, where he is now an Associate Professor of Physics.

He is a member of the American Institute of Physics, the Ass'n. of Physics Teachers and the Florida Academy of Sciences. In addition to serving on various university service committees, Dr. Marsh is involved in many civic activities. His special interest is an international folk dancing group which performs at university and community functions.

Richard C. Smith received a BS degree from Davidson College and his MS and PhD at Lehigh University (1966) prior to serving as a research physicist with the National Security Agency at Ft. George G. Meade, Maryland, from 1966-1968.

He is presently an Associate Professor of Physics at the University of West Florida, and was recently awarded a grant from the Ford Foundation Venture Fund for a "Personalized System of Instruction Project."

Dr. Smith's professional affiliations include the American Physical Society, American Ass'n. of Physics Teachers, Optical Society of America, Florida Academy of Sciences and the Society of the Sigma Xi. ●

1974 Calculator System Application Contest: Outside-U.S.A.

Congratulations go to Dipl. Ing. K. Vrana, Prague, Czechoslovakia, on winning the outside-U.S.A. branch of the 1974 Calculator System Application Contest. Mr. Vrana will receive his choice of one HP pocket calculator, either HP-45 or HP-80, as a prize. His application, which the panel of judges agreed almost unanimously was the most unusual entry, describes several techniques allowing blind people to use the HP 9820A and HP 9100A/B Calculators for musical as well as mathematical applications.

Sixty-six excellent entries from 23 countries were submitted in the outside-U.S.A. branch, making judging difficult and bringing the world total to 100 entries. We will publish as many entries in future *KEYBOARD* issues as time and space permit. Mr. Vrana's article will be included in Vol. 6 No. 6. To the other contestants, we give our sincere thanks for participating. We hope to give each of you the opportunity to take part again in the 1975 contest.

Shown below is a complete list of contest entries in the outside-U.S.A. branch, with titles and abstracts. These are arranged alphabetically by authors' names.

1. TRAFFIC ENGINEERING OF TLX AND TELEGRAPH STATIONS (9100B)
   by M. Agosthazi and Dr. G. Gosztony, Budapest, Hungary
   Article describes a system of 3 programs used to maximize availability of telex and telegraph stations in heavy traffic periods.
2. SPIROLATERALS AND ELLIPSOFLEX (9830A)
   by S. Balmer and F. Lombard, Aire-le Lignon, Switzerland
   These programs produce unique calculator art plots, using ideas from Martin Gardner's Mathematical Games column.
3. WAR PILOT'S DUEL (9100B)
   by Jan E. Blomquist, PhD, Sundsvall, Sweden
   This is a game for two players simulating an air fight between two airplanes visualized by plotting including ballistic graphs of projectiles.
4. GAME OF LIFE (9830A)
   by G. Bourquardez, Marignane, France
   This program prints out patterns on the 9866A in a field of up to 40 x 70. Pattern changes follow rules of the original Life game. Two versions.
5. LAGRANGE FUNCTION COMPUTATION (9830A)
   by G. Bourquardez, Marignane, France
   This program calculates and prints out Lagrange functions.
6. THE 9820A IN REACTOR PHYSICS (9820A)
   by F. C. DiFilippo and R. M. Waldman, Buenos Aires, Argentina
   This article describes use of the HP 9820A in a system to analyze pulsed neutron bursts acting on a two-core coupled system. Constants in the decay equation are calculated and corrections applied for reactivity of the source material and its corresponding error are calculated.
7. ON-LINE AIR VELOCITY MEASUREMENTS (9810A)
   by G. S. Ender, Dornbirn, Austria
   Program allows calculator to interpret data taken on-line by a thermal anemometer and dvm. Output gives the average effect of air refrigeration of the system on human skin.
8. VSWR PLOT (9810A)
   by Eric H. England, Shrivenham, Swindon, Wilts., England
   This program plots VSWR curves from experimental data obtained from HP 8755 or HP 8410. Also plots theoretical curves from estimated data.
9. COMPUTATION OF TABLES (9810A) (Realisation de Tableaux)
   by G. Fayet, Issoire, France
   Program allows calculating and printing out by columns the values of a function of complexity up to f (x,y,z) for specified ranges of the variables. (In French)
10. ANALYSIS OF GAMMA CAMERA IMAGING DATA USING A PROGRAMMABLE CALCULATOR (9820A)
    by Dr. David Feiglin, Melbourne, Australia
    This paper describes the use of a 9820A in a system with a gamma scintillation camera to trace the passage of a radioactive tracer through various organs of the human body, particularly kidney functions. Output can be seen on oscilloscope or plotted using 9862A Plotter.
11. CALCULATED DATA-SET STORAGE AND RECALL (9810A)
    by G. Gundersen, Grimstad, Norway
    Program uses a method of managing calculated data sets, particularly useful in surveying data. Each data set is assigned a number used to identify it for recall.
12. 3-DIGIT NUMBER GAME (9810A)
    by Molnar Gyula, Budapest, Hungary
    This program is a game for two players. The first player gives the calculator a 3-digit number. The second player must then guess the number.
13. GUIDED MISSLE GAME (9100A/B)
    by Nils Haglund, Eskilstuna, Sweden
    This program simulates dynamics of a first generation antitank missile. Operator guides missile by using digitizer cursor as joystick.
14. SIMULATION OF A DIGITAL COMMUNICATIONS LINK ON HP 9820A (9820A)
    by H. Peter Hartmann, Turgi, Switzerland
    This program simulates a digital communications link incorporating a convolutional coder at the transmitting end and a Viterbi decoder at the receiving end. The 9862A is used to represent the pertinent signals.
15. OXO-GAME (TIC-TAC-TOE) (9100A)
    by Prof. R. A. Hirvonen, Helsinki, Finland
    This program allows the user to play OXO against the 9100A.
16. ELEMENTAL FORMULA DETERMINATION IN HIGH RESOLUTION MASS SPECTROMETRY (9100B)
    by M. Jalobeanu, Cluj, Romania
    Given input of exact mass of an ion of unknown substance from mass spectrometer, programs determine all possible combinations of elements in substance.
17. SATPATH (9100B)
    by Donald Jender, Russell, Australia
    This program plots the ground path of an earth satellite.
18. PLANNING OF DIRECTIONAL DRILLING (9810A)
    by Arpad Kassay, Nagykanizsa, Hungary
    Program calculates angles and distances for deep well drilling where directions must change several times en route, such as offshore or urban area wells.
19. DESIGN OF ANCHORED BULKHEADS (9810A)
    by John Lankenau, Adelaide, So. Australia
    This program designs anchored bulkheads for waterfront use.

20. TEXT EDITING ON THE HP 9830A (9830A)
by Prof. Dr. Daniel Maeder, Geneva, Switzerland
This set of programs allows the operator to edit text using the 9830A.

21. BINOMIAL CURVE FIT (9100B)
by J. Massaut and P. Gillain, Seraing, Belgium
This program fits a binomial equation to a curve y=f(x) or a family of curves y=f(x,z).

22. TRAINING EXTRASENSORY PERCEPTION WITH THE 9820A (9820A)
by Dr. John F. C. McLachlan, Toronto, Ontario, Canada
This article describes a technique using 9820A to test an individual's level of clairvoyance, and to test whether the level can be improved by practice.

23. DATA TO MATRIX VIA PLOTTER (9830A)
by Ing. S. Milacek, Uvaly, Czechoslovakia
This program uses the plotter as a digitizer to enter and store elements (A_{ij}) in matrix array.

24. DIGITIZING VIA PLOTTER (9830A)
by Ing. S. Milacek, Uvaly, Czechoslovakia
This program uses the 9862A as a digitizer.

25. PARAPOWER (9810A)
by G. Miller, A. Plochocki, P. Mylinski, Warsaw, Poland
Used in thermoplastic processing, program converts capillary or rotational viscometry data into rheological parameters, then uses linear and parabolic (log) regressions to approximate the flow curves.

26. 9810A AS A NAVIGATIONAL AID (9810A)
by J. B. Miller, Hovik, Norway
This article describes a method of using the HP 9810A interfaced with a Motorola Mini Ranger System Mark III as a precision navigation system.

27. AERIAL TRIANGULATION IN PHOTOGRAMMETRY (9830A)
by Loi Poh Mun, Singapore
This program will perform models correction and strip adjustments on models coordinates observed with a stereo plotter to produce controls for small scale mapping.

28. LEAST SQUARES ADJUSTMENT TRIANGULATION (9830A)
by Loi Poh Mun, Singapore
This program produces a set of most probable values of coordinates in a network of triangulation or trilateration by adjusting observed angles or distances onto a few known fixed points.

29. STUDY OF EXPERIMENTAL DISTRIBUTION OF PROBABILITY (9830A)
by Emile Musyck, MOL, Belgium
This paper describes and gives examples of a method of reducing the systematic errors in an experimental curve of the distribution of probability.

30. CALCULATION OF ALCOHOL CONTENT IN ALCOHOLIC DRINKS (9810A)
by M. F. Nuijt, Haarlem, The Netherlands
This program calculates the twice-corrected density value (to 15 °C) of pure alcohol/water mixtures and from this value the alcohol content at 15 °C.

31. RAW MATERIAL ORDER SCHEDULING WITH 9830A (9830A)
by D. D. Parker, Brunico, Italy
This paper describes a set of programs which calculate and print out a monthly schedule of up to 600 raw material parts which acts as a working document, including monthly requirements, deliveries, and new orders for each part.

32. A PERSONAL LITERATURE REFERENCE INDEX (9830A)
by Dr. M. C. Patterson, Hamilton, Ontario, Canada
This program stores and retrieves information from a literature reference index based upon the feature card principle of indexing a collection of literature references.

33. "21"GAME (9100A/B)
by Capt. John Plaxton, Winnipeg, Manitoba, Canada
This program allows a human opponent to play the calculator in a simulated card game of Twenty-one.

34. HIGH-LOW GAME (9100A/B)
by Capt. John Plaxton, Winnipeg, Manitoba, Canada
This game can be played solitaire or between two players. The object is to roll five "dice" to get as high or low a score as possible.

35. SLOT MACHINE (9100A/B)
by Capt. John Plaxton, Winnipeg, Manitoba, Canada
This program simulates a slot machine. Calculator determines the payout and prints a cumulative total of winnings or losses.

36. FLOWCHART PLOT (9830A)
by Peter Post, Marburg, Germany
This program plots flowcharts for publication from data input from a rough copy. Corrections can be made.

37. PRINTED CIRCUIT DRAWING AND PRODUCTION (9100B)
by J. Radnai and T. Tanczos, Budapest, Hungary
These programs allow the 9125A/B Plotter to draw printed circuits either on oversize paper for photographing, or actual size on copper-coated board with special ink, ready for etching.

38. WHEN A 9820A MAKES MISTAKES (9820A)
by J. D. Ralphs, Hanslope, Milton Keynes, England
Describes use of 9820A in simulation of communication link, including generation of signals, intentional introduction of random errors, and output error analysis.

39. COUNT (9100B)
by Dipl. Ing. W. Reuter, Maintal, Germany
This program permits given numbers to be described in arbitrary numerical quantities. (In German)

40. TIME INDICATOR WITH SIGNAL (9100B)
by Dipl. Ing. W. Reuter, Maintal, Germany
This program can be set like an alarm clock to give an audible indication (plotter ticks 6 times) when the desired time is reached.

41. PRIME NUMBER QUARTET SEARCHER (9100B)
by Dipl. Ing. W. Reuter, Maintal, Germany
This program searches for all prime number quartets up to 10^{11}.

42. BILLIARD GAME (9100B)
by Dipl. Ing. W. Reuter, Maintal, Germany
This program simulates a billiard game. The "ball" slows down and eventually comes to rest. Number of times ball struck is displayed. Path of ball is plotted.

43. LOTTO (9100B)
by Dipl. Ing. W. Reuter, Maintal, Germany
This program is a game corresponding to Lotto, in which the user inputs a number less than 1 and the program generates six random numbers of value between 1 and 49 which are then displayed.

44. DASHED AND DOTTED CURVE PLOT (9100B)
by Dipl. Ing. W. Reuter, Maintal, Germany
Plots mathematical functions with dashed, dotted, or combined dash-dotted lines with several element formats. (In German). Improvement of program 09100-70091.

45. MINIMUM WALL THICKNESS OF OUTSIDE-LOADED TUBE (9100B)
by Dipl. Ing. W. Reuter, Maintal, Germany
This program calculates for a tube with an outside load the minimum wall thickness required so that no radial deflection will occur. (In German)

46. DAY OF WEEK OF A GIVEN DAY (9810A)
by Dr. Ing. M. Ruggieri, Cantu' (Milano) Italy
Given input of number of month, day, and year, program prints out day of week. Entry errors are indicated.

47. APERIODIC SEQUENCE OF PSEUDORANDOM NUMBERS WITH NONLINEAR FEEDBACK (9810A)
by Dipl. Ing. Jan Sandtner, Waldenburg, Switzerland
This program generates the aperiodic sequence of pseudorandom numbers within the interval (0,1), each number consisting of ten significant digits.

48. 9820A AIDS TO TEACH PROBABILITY THEORY (9820A)
by J. Slavik and K. Vavruska, Zilina, Czechoslovakia
This paper describes a series of programs designed for easy student operation to teach probability theory. Results are plotted to enhance understanding.

49. 9830A AS A SIMULATOR (9830A)
by J. Slavik & K. Vavruska, Zilina, Czechoslovakia
This paper describes use of the 9830A as a simulator of transport and communication systems such as road intersections, telephone exchanges, and queueing systems.

50. DAY NAME OF A GIVEN DATE (9830A)
by C. St. Gurau and L. Stan, Savinesti, Romania
Give a date in numerical form (YYMMDD), this program displays the name of the day.

51. ENVIRONMENTAL ANALYSIS AND SIMULATION WITH 9810A (9810A)
by Dr. Milan Straskraba, Sluknovska, Czechoslovakia
This article is based on environmental studies using the 9810A to model and analyze pollution conditions in various water bodies based on climate, pollution, algae growth, geographic environmental variables, etc.

52. VALUES OF E AND PI (9810A)
by Marcel Sutter, Oberwil, Germany
These programs compute the values of e and pi. The output has at least 250 decimals. (Text in German)

53. AUSWERTUNG VON KONIMETERPROBEN MIT HILFE DES QUANTIMET 720 (INTERPRETATION OF KONIMETER PROBE DATA WITH HELP OF QUANTIMET 720) (9810A)
by Ing. Songott & Dipl. Ing. Otti, Einödmayergasse, Austria
This article describes use of the HP 9810A on-line with the Quantimet 720 to interpret and output, via the Facit typewriter, measurement data of disease-producing dust particles. (Text in German)

54. RUNGE-KUTTA SOLUTION TO 1ST AND 2ND ORDER DIFFERENTIAL EQUATIONS (9810A)
by Dr. Kurt Tanner, Immensee, Switzerland
This program solves first and second order differential equations by the method of Runge-Kutta.

55. CLINICAL EEG ANALYSIS SYSTEM (9810A)
by Doug Teeple, Hamilton, Ontario, Canada
This program provides a qualitative plot and a quantitative printout of frequency information derived from electroencephalographic data for fast and accurate interpretation for the clinician.

56. LOGIC NETWORK ANALYSIS (9100B)
by Zoltan Vajda, Budapest, Hungary
This program calculates response of a combinational logic circuit to a given combination of up to 10 inputs (and unlimited outputs) or sweeps the input from all 0's to all 1's, and prints out the complete truth table.

57. ALCUIN'S GAME (9100A/B)
by J. Th. N. Vellenga, Zoetermeer, The Netherlands
This program allows the user to play the wolf-goat-cabbages game. User is challenged to get all three items across a river, one at a time, without damage to any item.

58. MATCHSTICK GAME (9830A)
by M. Venkatachalam, Madras, India
This program is a game played by two people. 15 matchsticks are arranged in three groups; players in turn pick a number of matchsticks from one of the piles. Player picking up last one loses. One player can play against the 9830.

59. NOMOGRAM PLOTTING (9100A/B)
by M. Venkatachalam, Madras, India
Program plots a nomogram for any formula having the form $y = k\,u^m v^n w^p \ldots$, where y, u, v, w... are variables and k, m, n, p... are constants. One example is for volume of cylinder in cc for given ranges of diameter and height.

60. INTEGRATED CIRCUIT LAYOUT DESIGN WITH HP DESKTOP CALCULATORS (9100A/B)
by A. Vladimirescu and D. Prisecaru, Bucharest, Romania
Set of programs enables layout design, drawing, verification, and digitizing of complex integrated circuits.

61. MAP PROJECTIONS (9820A)
by M. Vlietstra, Johannesburg, So. Africa
This program allows plotting maps according to either Mercator or modified projections. Program allows rotating the earth's latitude and longitude grids and making projections on the new grids.

62. HP CALCULATORS USED BY THE BLIND (9820A, 9100A/B)
by Dipl. Ing. K. Vrana, Prague, Czechoslovakia
Techniques allow blind user to read in Braille the x, y, and z 9100A/B registers; to transform numerical data into coded audio tones; and to transform inputted Braille musical notation into standard black and white notation.

63. ELECTRIC CIRCUIT DIAGRAM PLOTTING (9100A/B)
by Dr. V. Vuckovic, Belgrade, Yugoslavia
Series of programs enables plotting electric and other complete circuit diagrams including resistors, inductors, diodes, transistors.

64. FINITE ELEMENT ANALYSIS OF CYLINDRICAL SHELLS (9820A)
by J. Vykutil, Brno, Czechoslovakia
Program calculates and prints out on typewriter the deflection and other characteristics of loaded cylindrical shells. The finite element method is used.

65. AUTOMATED SYSTEM FOR MEASURING MAGNETIC MATERIALS (9100B) or (9810A)
by Dr. Friedrich Walz, Stuttgart, Germany
This article describes a completely automated method for measuring the magnetic aftereffect in ferro- and ferrimagnetic materials using the HP 9100B and 9810A calculators.

66. CONTINUOUS SYSTEM SIMULATOR (9830A)
by Dr. A. Watson, Bristol, England
This paper describes a system of programs which simulates an electrical analogue of a mechanical or other problem involving differential equations. Understanding and operation of programs is easy for unskilled user. ■

THE Crossroads

THE EIGEN-SOLUTION OF LINEAR SYSTEMS

by James N. Shapiro

Sooner or later almost everyone who uses any kind of programmable calculator or computer is faced with the problem of finding the solution to a set of linear equations. If the number of equations is very small, like three or four, Cramer's rule may be applied. For larger systems the systematic elimination of unknowns by the Gauss-Jordan or Gauss-Seidel techniques is often used. Any of these methods, properly applied, will give the solution to a "well posed" set of linear equations. By "well posed" we shall mean a system in which the number of equations and unknowns are equal and for which the determinant of the (square) coefficient matrix $\bar{\bar{A}}$ is non-zero. If our system of equations looks like

$$a_{11}y_1 + a_{12}y_2 + \dots + a_{1N}y_N = b_1$$
$$a_{21}y_1 + \quad \dots\dots \quad + a_{2N}y_N = b_2$$
$$\vdots$$
$$a_{N1}y_1 + \quad \dots\dots \quad + a_{NN}y_N = b_N ,$$

we may write it in a shorthand notation as

$$\bar{\bar{A}} \cdot \bar{y} = \bar{b} .$$

The components of the vector $\bar{b}$ are, in order, the numbers which reside on the right hand side of the equality in each equation. For all but instructional examples the vector $\bar{b}$ is a measured quantity subject to errors of measurement. The components of the vector $\bar{y}$ are, in order, the unknowns, which we wish to determine. The elements of the matrix $\bar{\bar{A}}$ are the coefficients a_{ij} arranged exactly as above.

Unfortunately, most of us are introduced to linear equations by means of instructional examples. We are instructed to find the solution to a certain set of equations. Both the coefficients, $\bar{\bar{A}}$, and right hand side, $\bar{b}$, are given with absolute accuracy. If the system is well posed there is only one solution and after some effort either by us or by a calculator, or, more likely, by both, we find the solution. As long as all of our input numbers can be relied on as exact, the problem becomes one of instructing our calculator to perform certain operations in a certain sequence. If we make the program general so that systems of all sizes up to the capacity of the machine can be handled, we can solve all systems which fit into our machine and we need worry no more about programming the solution to exact linear systems.

In the real world, however, measurements are never exact. Let's look at systems for which the coefficient matrix is known exactly (as it usually is) but for which the vector $\bar{b}$ consists of measured, i.e., inexact, quantities. As a final restriction let's direct our attention to systems for which the matrix, $\bar{\bar{A}}$, is symmetric. By a symmetric matrix we mean one in which the matrix remains unchanged if we interchange the rows and columns. The matrix

$$\bar{\bar{A}} = \begin{bmatrix} -1 & 3 \\ 3 & 10 \end{bmatrix}$$

is symmetric.

This last requirement may sound very restrictive to one who is not familiar with linear systems, but, in fact, this is not the case. Many systems are already symmetric, for example the coefficient matrix which results from fitting a set of data with a least squares polynomial. Non-symmetric systems can always be made symmetric by multiplying both sides of the equation

$$\bar{\bar{A}} \cdot \bar{y} = \bar{b}$$

by $\bar{\bar{A}}^T$ where $\bar{\bar{A}}^T$ is simply $\bar{\bar{A}}$ transposed so that rows become columns and vice versa. For example if $\bar{\bar{A}}$ is

$$\bar{\bar{A}} = \begin{bmatrix} 2 & 7 \\ 3 & -6 \end{bmatrix} ,$$

$\bar{\bar{A}}^T$ is

$$\bar{\bar{A}}^T = \begin{bmatrix} 2 & 3 \\ 7 & -6 \end{bmatrix} .$$

(The interested reader can verify that the product $\bar{\bar{A}}^T \cdot \bar{\bar{A}}$ is symmetric in this case). After multiplying both sides by $\bar{\bar{A}}^T$ we have

$$\bar{\bar{C}} \cdot \bar{y} = \bar{b}'$$

where

$$\bar{\bar{C}} \; (= \bar{\bar{A}}^T \cdot \bar{\bar{A}})$$

is a symmetric matrix, and

$$\bar{b}' \; (= \bar{\bar{A}}^T \cdot \bar{b})$$

is a new right hand side.

If a (symmetric) system

$$\bar{\bar{A}} \cdot \bar{y} = \bar{b}$$

is to be solved for $\bar{y}$ when $\bar{b}$ consists of measured numbers we shall be concerned with the sensitivity of the components of the solution vector, $\bar{y}$, to variations in the components of the data vector, $\bar{b}$. While offhand we might expect that small relative changes in the vector $\bar{b}$ could only produce small relative changes in $\bar{y}$ the following example should convince us that this is not at all the case. The system

$$\frac{116}{17} y_1 + \frac{396}{17} y_2 = 100$$
$$\frac{396}{17} y_1 + \frac{1601}{17} y_2 = 400 \qquad (1)$$

can easily be solved. The solution is

$$\bar{y} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} .$$

But if we change $\bar{b}$ only slightly to

$$\bar{b}' = \begin{bmatrix} 99 \\ 401 \end{bmatrix}$$ the new solution vector

becomes $\bar{y} = [-.1747, 4.301]$. Try it, if you have a program for solving linear equations, or just substitute the solutions into the equations to verify them.

This is a most discomforting state of affairs. A change of 1% in one of the components of $\bar{b}$ and of ¼% in the other has drastically altered the solution vector. In order to understand what is going on here we will need a brief discussion of eigenvalues and eigenvectors.

Editor's Note: Dr. Shapiro, who is acting as guest editor of The Crossroads this time, has had other articles published in *KEYBOARD*. See Vol. 4. No. 2, p. 13, for his curriculum vitae.

The eigenvectors occupy a very special position in a linear system. These vectors are completely unrelated to the data vector, $\bar{b}$, or the solution vector, $\bar{y}$. They depend only on the coefficient matrix $\bar{\bar{A}}$. The property which defines the eigenvectors $\bar{u}_i$, i = 1, 2, ... N (where N is the number of equations) is that when any one is multiplied by $\bar{\bar{A}}$ the vector produced is parallel to the original (unmultiplied) vector. In mathematical notation

$$\bar{\bar{A}}\cdot\bar{u}_i \propto \bar{u}_i \ .$$

The proportionality factor is the associated eigenvalue λ_i, i.e., $\bar{\bar{A}}\cdot\bar{u}_i = \lambda_i\bar{u}_i$. It is customary and convenient to normalize the eigenvectors to unity and to designate the normalized unit vectors with a ^. Then the eigenvalue equation above becomes

$$\bar{\bar{A}}\cdot\hat{u}_i = \lambda_i\hat{u}_i \ , \qquad i = 1, 2, \ldots, N \ ,$$

for each of the N eigenvectors and their associated eigenvalues. Aside from certain pathological matrices called defective, we can find N eigenvectors $\hat{u}_i$. It is easy to show that the dot product of any two eigenvectors corresponding to different eigenvalues is zero. (When two or more eigenvalues are equal this is called degeneracy. In such cases we can find corresponding eigenvectors with zero dot product but they are no longer unique.) But we remember from elementary vector analysis that if the dot of product of two (non-zero) vectors is zero the vectors are orthogonal or perpendicular. We can illustrate this concept very easily with the following 2-dimensional example.

Let the coefficient matrix be given by

$$\bar{\bar{A}} = \begin{bmatrix} 5/4 & -\sqrt{3}/4 \\ -\sqrt{3}/4 & 7/4 \end{bmatrix} . \qquad (2)$$

It is easy to show that the eigenvectors and eigenvalues are

$\lambda_1 = 1$, $\hat{u}_1 = (\sqrt{3}/2, 1/2)$, and

$\lambda_2 = 2$, $\hat{u}_2 = (-1/2, \sqrt{3}/2)$. Note that $\hat{u}_1\cdot\hat{u}_2 = 0$ so that they are indeed orthogonal. The $\hat{u}_i$'s being of length one and mutually orthogonal make a convenient set of coordinate axes. Let's use them as such.

We are now in a position to present a simple geometrical method for solving symmetric linear systems. We shall not derive our method, although it is not hard, particularly for those familiar with dyadic notation.

SOLUTION TO LINEAR SYSTEMS

I First find the component of the right hand side, $\bar{b}$, along each of the eigenvectors (our coordinate axes) i.e., $\bar{b}\cdot\hat{u}_i$, i = 1, 2, ..., N ,

II Divide each of these components by the corresponding eigenvalue, i.e.,

$$\bar{b}\cdot\hat{u}_i/\lambda_i, \ i = 1, 2, \ldots, N \ ,$$

III To find the j'th component, y_j, of the solution vector, $\bar{y}$, multiply the i'th answer from II by the j'th component of $\hat{u}_i$ for all i and sum, i.e.,

$$y_j = \sum_{i=1}^{N} (\bar{b}\cdot\hat{u}_i/\lambda_i)\,(\hat{u}_i \text{ j'th component}) \ .$$

No doubt the above procedure sounds complicated right now, so let's go through it with some numbers. Using the coefficient matrix, $\bar{\bar{A}}$, of Eqn. 2 we can try our method on the simple linear system

$$5y_1 - \sqrt{3}y_2 = 8$$
$$-\sqrt{3}y_1 + 7y_2 = 16 \ .$$

Here we have supplied $\bar{b}$:

$$\bar{b} = (2, 4)$$

I Components of $\bar{b}$ along eigenvectors

$$\bar{b}\cdot\hat{u}_1 = (2, 4)\cdot(\sqrt{3}/2, 1/2) = 2 + \sqrt{3}$$
$$\bar{b}\cdot\hat{u}_2 = (2, 4)\cdot(-1/2, \sqrt{3}/2) = -1 + 2\sqrt{3}$$

II Division by eigenvalues

$$\bar{b}\cdot\hat{u}_1/\lambda_1 = 2 + \sqrt{3}$$
$$\bar{b}\cdot\hat{u}_2/\lambda_2 = -1/2 + \sqrt{3}$$

III Solution by summation

$$y_1 = (2 + \sqrt{3})(\sqrt{3}/2) + (-1/2 + \sqrt{3})(-1/2) = 7/4 + \sqrt{3}/2$$
$$y_2 = (2 + \sqrt{3})(1/2) + (-1/2 + \sqrt{3})(\sqrt{3}/2) = 5/2 + \sqrt{3}/4$$

The solution has been obtained by summing, for each component, a term inversely proportional to each eigenvalue. Before we elaborate on this feature, it is possible to state very simply, in geometrical terms, the operations which we have performed. Looking at Figure 1, $\hat{u}_1$ and $\hat{u}_2$ have been assigned (perpendicular) directions in space. But, implicit in the whole development above, and implicit in any operation with matrices or vector components is a certain coordinate system. We can determine the orientation of this coordinate system by examining the components of $\hat{u}_1$ and $\hat{u}_2$ in this system. In the present case it is easy to show that the implicit coordinate system x_1, x_2 is rotated 30° clockwise from the eigenvector ($\hat{u}_1$, $\hat{u}_2$) system (see Figure 1).

The solution of any linear system is equivalent to a coordinate transformation and a division. We are (unfortunately) given the problem in some non-eigenvector coordinate system. We first find the components of the right-hand side in the eigensystem. We then divide these components by their respective eigenvalues. Finally we transform the divided components back into the original system.

All of this sounds like a lot of work to obtain the solution to a simple set of equations, and indeed it is if one is interested in the solution only when the right hand side is exact. When this is not the case, an eigenvalue analysis can yield information unobtainable in any other way. The crucial parameters are the relative error in the measurements and the so-called condition number of the system. The condition number is the ratio of the largest to smallest eigenvalue. Since we divide each of the components of $\bar{b}$ in the eigensystem by an eigenvalue, those components in the direction of eigenvectors having small eigenvalues will be magnified greatly. (In fact, the condition of zero determinant is nothing more than a statement that at least one of the eigenvalues is zero!). If the smallest eigenvalue is a small fraction of the largest one, (another way of saying that the condition number is large), and if there is error in the components of $\bar{b}$, then it is entirely possible for the error to be magnified to such an extent that the true solution becomes swamped by the error. Let's use the example from Eqn. 1 with

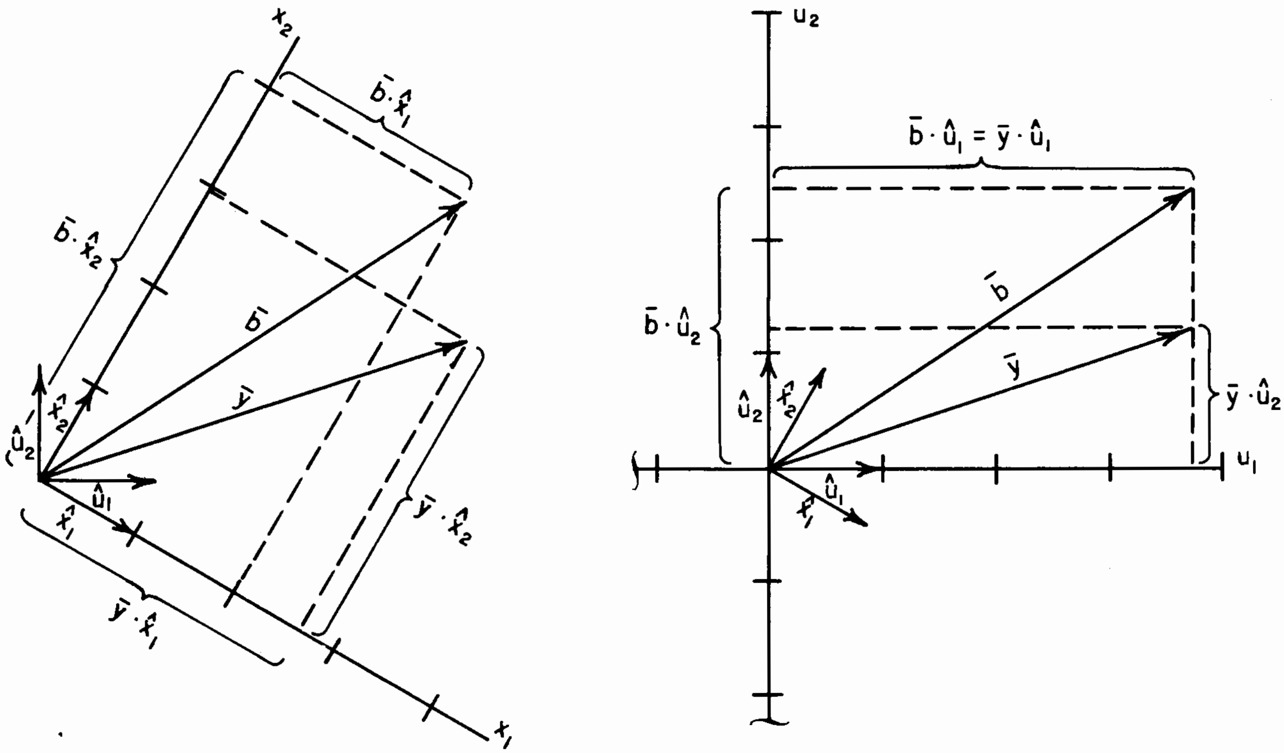
$$\bar{\bar{A}} = \begin{bmatrix} 116/17 & 396/17 \\ 396/17 & 1601/17 \end{bmatrix}$$

to illustrate this point. We have already shown that when $\bar{b} = (100, 400)$, $\bar{y} = (1, 4)$ and when $\bar{b}$ is changed "slightly" to $\bar{b} = (99, 401)$, $\bar{y}$ changes "drastically" to $\bar{y} = (-.1747, 4.301)$. Suppose that, in fact, the true value of the right-hand side is $\bar{b} = (100, 400)$ but that our measurements are accurate to only 1%. Then the right-hand side $\bar{b} = (99, 401)$ is a possible result of our measurements. The three-step eigenvalue analysis reveals the problem. If we call $\bar{y}\lambda_i$ the part of the solution due to the i'th eigenvalue (i = 1 or 2 in this case) we find

$$\bar{y} = \bar{y}\lambda_1 + \bar{y}\lambda_2 = (1703/1700)(1, 4) + (5/17)(-4, 1)$$

where the first term is due to the component of $\bar{b}$ along $\hat{u}_1 = (1, 4)/\sqrt{17}$ with $\lambda_1 = 100$, and the second term is due to the component of $\bar{b}$ along $\hat{u}_2 = (-4, 1)/\sqrt{17}$ with $\lambda_2 = 1$. What has happened is that the second eigenvalue (1) is so small compared with the first (100) that the error (and true solution) arising from the component of $\bar{b}$ along $\hat{u}_2$ has been magnified by 100 compared with the true solution (and error) arising from the other component of $\bar{b}$. The magnified error pollutes (in general) all of the components of $\bar{y}$. We would have been better off if we had approximated $\bar{y}$ by $\bar{y}\lambda_1 = (1703/1700)(1, 4) = (1.00, 4.01)$, which is very close (~1%) to the correct solution $\bar{y} = (1, 4)$.

A general criterion for solving linear systems now emerges. We should for consistency neglect the contribution to the solution which arises from those small eigenvalues λ_i, for which λ_i/λ max $\lessapprox$ relative error.



In x_1, x_2 coordinate system

$\bar{b}$	$\bar{y}$
$\bar{b}\cdot\hat{x}_1 = 2$	$\bar{y}\cdot\hat{x}_1 = y_1 = 7/4 + \sqrt{3}/2$
$\bar{b}\cdot\hat{x}_2 = 4$	$\bar{y}\cdot\hat{x}_2 = y_2 = 5/2 + \sqrt{3}/4$

In u_1, u_2 coordinate system

$\bar{b}$	$\bar{y}$
$\bar{b}\cdot\hat{u}_1 = 2 + \sqrt{3}$	$\bar{y}\cdot\hat{u}_1 = \bar{b}\cdot\hat{u}_1/\lambda_1 = 2 + \sqrt{3}$
$\bar{b}\cdot\hat{u}_2 = -1 + 2\sqrt{3}$	$\bar{y}\cdot\hat{u}_2 = \bar{b}\cdot\hat{u}_2/\lambda_2 = -.5 + \sqrt{3}$

Figure 1 — The data and solution vectors $\bar{b}$ and $\bar{y}$, and their components in the original (x_1, x_2) and eigen (u_1, u_2) coordinate systems.

This is an interesting example. With data of limited accuracy, not only is an exact solution unnecessary, but it can magnify the error to such an extent as to completely mask the true solution.

We hope to have pointed out some of the pitfalls of applying mathematically exact procedures to physical data. Admittedly, the eigenvalue analysis of a linear system takes more time and trouble than any of the more common techniques. But, it always gives at bare minimum the same solution as other methods, and often provides a framework in which one can observe the effects on the solution of error in the data. A good analyst will use an eigenvalues analysis to remove those components of the solution vector in which the error or noise is comparable to the signal, thus enhancing the signal to noise ratio.

We have only touched the field of eigenvalue analysis. We addressed ourselves only to symmetric linear systems with known eigenvalues and eigenvectors. Methods for finding the eigenvalues and vectors can be found in almost any textbook on applied mathematics. I recommend Lanczos (1964). Non-symmetric systems can also be attacked by methods similar to those presented here. The most interesting cases, those in which the number of equations is either greater than or less than the number of unknowns can also be studied in terms of eigensystems as can most non-linear systems of equations. I refer the interested reader to the third chapter in Lanczos (1961).

REFERENCES

Lanczos, C., *Linear Differential Operators*, Van Nostrand, New York, 564 pp., 1961.

Lanczos, C., *Applied Analysis*, Prentice-Hall, Englewood Cliffs, New Jersey, 539 pp., 1964.

programming tips

MULTIPLE BRANCHING (9810A)

Prof. Benjamin B. Ross, University of Oregon Medical School, Portland, Oregon, submitted this technique for increasing the number of branching options in the HP 9810A after a stop for data entry.

Branching options may be increased and erroneous branching avoided if 10^{99} is placed in the x and y registers of the 9810A prior to a STOP for data entry and followed by a conditional branch instruction. This permits branching to two different subroutines using the IF FLAG and IF x = y options. If zero is not a real data entry, a third branch option may be added by clearing the x register, multiplying and asking again if x = y. However, if zero is a valid entry (e.g., time = 0), branching out of the subroutine may be accomplished by "IF x = y" and the flag may be used for other purposes, such as branching to an error correction routine. In this latter case, any number (except 10^{99}) entered will cause the program to continue normally.

LAW OF COSINES (9100A/B, 9810A)

John G. Langdon, Anaheim, California, kindly submitted this short sequence for solving the law of cosines, which works for the HP 9100A/B or the HP 9810A.

Given two sides of a triangle, a and b, and the included angle C, find the third side, c. With a, C and b entered in the s, y and z registers, the sequence is:

TO RECT
ROLL ↑
—
↓
TO POLAR.

The length of the third side, c, appears in the x register.

Referring to Fig. 1, the first step calculates x_1, y_1; the next steps determine $x_2 = x_1 - b$; the final step calculates c from x_2, y_1.

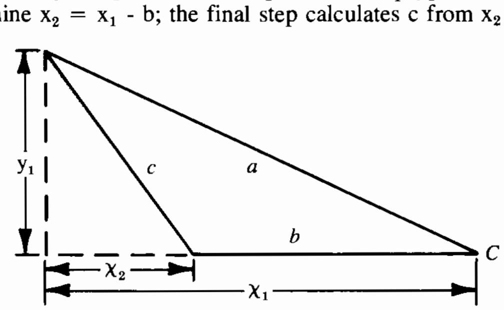


Fig. 1 Triangle Nomenclature

Of course, a and b can be interchanged in the program. Sometimes x_1 is negative, but this does not affect the conversion to get side c.

ROUNDING TO NEAREST FRACTIONS (9810A)

Our thanks go to Richard P. Demme, Carol City, Florida, for this programming tip which uses the 9810A Calculator with a math block in slot one.

Often when an engineer converts from hand calculations to computer calculations he encounters round-off problems. One such problem is rounding to the nearest fractional inch. As an example, engineering design involving large structures may require accuracy to the nearest ⅛ inch. Using the rounded numbers, the designer proceeds to make further calculations.

The short routine below calls for entry of decimal inches in the x-register, rounds to the nearest ⅛ inch, then displays the rounded fractional number in the x-register. Entries may be either positive or negative.

With appropriate modification, this short routine may be used to round any decimal number to any nearest fractional number.

LISTING

```
0000--CLR---20
0001-- 1 ---01
0002--STP---41
0003-- UP---27
0004-- 8 ---10
0005-- X ---36
0006-- 0 ---00
0007-- K ---55
0008-- 9 ---11
0009-- UP---27
0010-- 8 ---10
0011--DIV---35
0012--XEY---30
0013--END---46
```

EXAMPLE

```
INPUT
          0.437*
OUTPUT
          0.375

INPUT
          0.438*
OUTPUT
          0.500

INPUT
     -111111.687*
OUTPUT
     -111111.625
```

STRING COMPARISONS (9830A)

Francois Martin of the Tudor Engineering Company, Seattle, Washington, called to our attention a problem sometimes encountered in 9830A programs using string variables.

When a "yes" or "no" reply is input in answer to a program query and the reply is tested to determine program branching, as in the line:

130 IF B$(1,1) = "Y" THEN 900

the user normally types in Y or YES without pressing the SHIFT key.

This is the expected reply, resulting in a "true" decision in comparing the Y's and proper branching to line 900, provided the Y in quotation marks above was also programmed in the unshifted mode. However, if the programmer held the shift key down while typing the Y in line 130, the test then compares Y (octal code 131) with y (octal code 171), so branching will not occur. The same difficulty occurs if the programmed Y was entered in the unshifted mode and the user inadvertently enters his reply in the shifted mode.

In all 9830A programs published by HP, alpha string characters are entered in the unshifted mode. An alpha YES reply in the unshifted mode then causes the expected program action. Users should remember that although the display and the printed 9866A output look the same for either shifted or unshifted letters, the calculator sees and compares different octal codes.